Journal of the
OPTICAL SOCIETY
of AMERICA

VOLUME 34, NUMBER 12 DECEMBER, 1944

Principles of Microscope Illumination and the Problem of Glare

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(Received September 20, 1944)

The manipulations in using the microscope, other than the selection of lenses, focusing, and placement of slides, relate to the illumination. When the illumination is not handled with judgment, glare and lack of contrast result, and crisp images are not obtained. Effective illuminations depend equally upon three factors, the condenser, the field stop, and the illuminant proper. The adjustments for superior glare-free illumination are discussed and the term "controlled" illumination is suggested as an appropriate designation for such illumination. Aspects of the lighting requirements of the microscope are analyzed, and it is shown that suitable regulation of the apertures of the field and condenser stops are the basic requirements for controlled illumination. Various types of lighting may be used in conjunction with these adjustments. The paper is presented against a background of the pertinent historical developments and the various factors relating to illumination are reviewed in a broader fashion than is common in texts on microscopy. Explanations of the principles and the defects of illumination are presented with the general biological user of the microscope in mind. The shapes and proportions of the various illuminating beams that play on specimens are illustrated. Aberrations and the origin of glare are explained. In general, there are two extremes in the practical use of illumination—illumination for high resolution and illumination for maximum contrast. The microscopist should ordinarily vary his illumination as he works to obtain the advantages of each type of illumination. The conditions of each are fully treated.

THOUGH the basic principles of microscope illumination, and effective systems of illumination too, were developed a half-century ago, it is apparent that these principles have not been sufficiently well understood to guide current practice in microscopy. Most microscope users have consistently ignored the better methods of illumination and have tolerated discomfort, inadequate illumination, and work that does not reflect the capacity of a precision instrument. Recent texts[1] on microscopy and photomicrography have uniformly emphasized the importance of illumination, and the half-century old techniques have been presented anew as adjuncts important to successful work with the microscope. Even so, the practical worker has not yet been provided with a historical perspective nor with a basis for a clear understanding of effective microscope illumination. The treatment below is an essay in this direction.[2]

[1] John Belling, *The Use of the Microscope* (McGraw-Hill Book Company, Inc., New York, 1930). S. H. Gage, *The Microscope* (Comstock Publishing Company, Ithaca, New York, 1932). Conrad Beck, *The Microscope, Theory and Practice* (R. and J. Beck, London, 1938). F. J. Muñoz and H. A. Charipper, *The Microscope and Its Use* (Chemical Publishing Company, Brooklyn, New York, 1943). J. E. Barnard and F. V. Welch, *Practical Photomicrography* (E. Arnold and Company, London, 1936). G. G. Reinert, *Praktische Mikrofotografie* (W. Knapp, Halle, 1937). R. M. Allen, *Photomicrography* (D. Van Nostrand Company, New York, 1941). C. P. Shillaber, *Photomicrography in Theory and Practice* (John Wiley & Sons, Inc., New York, 1944).

[2] I am greatly indebted to Dr. O. W. Richards, Spencer Lens Company, and Dr. G. L. Walls, Bausch and Lomb Optical Company, for their critical comments on a preliminary draft of the manuscript.

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A variety of illuminating devices for microscopy with transmitted light has been introduced since the early days of the nineteenth century.[3] The modern illuminating apparatus, however, stems from the condenser of Abbe. When this device was introduced,[4] Abbe stated that for general microscopical work, if one focused the image of a cloud (or of an artificial light source behind a globe-like flask of water, preferably tinted blue) upon an object in the field of view, suitable illumination was provided. Although the concept of focused illumination antedates Abbe by many years, most of the early advocates of focused illumination compromised and thus did not employ it in practice.[5]

For a decade after the introduction of the Abbe condenser, during which period the corrected substage condenser also appeared,[6] the device gradually came into scientific use. The rise of bacteriology and of the more advanced microscopical anatomy of the time facilitated the acceptance of the condenser as an integral part of the microscope. The condenser, however, was a complication and how it was to be used became a problem. Abbe's experiments leading to a theory of microscope image formation[7] were based on the use of narrow illuminating beams (i.e., constricted condenser stop) and, often, of obliquely directed narrow beams. "According to theory, wide cones are not as good as narrow."[8] A number of practical workers followed Abbe in the use of narrow cone illumination.[9] In the hands of the majority of workers, however, the condenser became merely a device for modifying the intensity of illumination. This was done by opening or constricting the condenser iris or by altering the condenser focus through raising or lowering the condenser level. Similar misuse of the condenser is common today.[10]

Nelson, during the 1880's and later, repeatedly demonstrated[11] that if wide-angled illuminating cones from the condenser, associated with a wide condenser aperture, were focused on an object, finer details of structure could be seen than when narrow beams were employed. He showed that the highest degree of resolution that is possible for a given objective and a given wave-length of light is obtained only when the cone of condenser illumination is exactly as wide-angled as the widest cone that may enter the objective. With such illumination, according to the analytical treatment of Rayleigh,[12] an object acts as if it were self-luminous, and it is a self-luminous object that allows the highest resolution possible for a given objective. Resolution, in fact, is just twice as great as that obtained by a parallel beam and an object which acts like a grating. Intermediate degrees of resolution are obtained when the condenser stop is constricted to permit less than full cone illumination. It is ordinarily stated[13] that resolution is proportional to one-half of the N.A.[14] value of the objective plus one-half the working (N.A.) aperture of the condenser.

Published photographs illustrating the effect of various condenser apertures on the resolution of an objective seem to deal only with transparent or refractile objects like diatoms;[15] gratings, too, have been common as test objects. Figure 1

[3] Hugo von Mohl, *Mikrographie, oder Anleitung zur Kenntniss und zum Gebrauche des Mikroskops* (L. F. Fues, Tübingen, 1846). E. M. Nelson, J. Roy. Microscop. Soc. **7**, 90–105 (1891).

[4] E. Abbe, Arch. f. mikroskop. Anat. **9**, 469–480 (1873). Reprinted in *Gesammelte Abhandlungen von Ernst Abbe* (G. Fischer, Jena, 1904), Vol. 1, and condensed translation in E. Abbe, Monthly Microscop. J. **13**, 77–82 (1875).

[5] Nelson, reference 3. Focused illumination is but one of the possible techniques for securing effective illumination; it lends itself well to control. Control is the *sine qua non* of good illumination and, as will be shown, control is possible only when the plane of condenser focus is adjusted so that it falls at or near the level of objects (on a slide) in the field of view.

[6] Note in J. Roy. Microscop. Soc., p. 609 (1889).

[7] E. Abbe, Arch. f. mikroskop. Anat. **9**, 413–468 (1873). Also in *Gesammelte Abhandlungen von Ernst Abbe*, Vol. 1, pp. 45–100.

[8] E. Abbe, J. Roy. Microscop. Soc., pp. 721–724 (1889).

[9] The distinction between wide and narrow cones is shown in Fig. 6.

[10] It should be stated emphatically at the outset that illumination intensity is a most important variable in microscopy; and where serious microscopy is attempted, a rheostat, neutral wedges, or filters should be employed to regulate intensity. The condenser is not for this purpose.

[11] E. M. Nelson, Eng. Mech. **40**, 68, 157–158, 263, 282 (1884). E. M. Nelson, J. Roy. Microscop. Soc., pp. 282–289 (1910). Nelson, reference 3.

[12] Lord Rayleigh, Phil. Mag. **42**, [5], 167–195 (1896).

[13] L. C. Martin, *An Introduction to Applied Optics* (Isaac Pitman and Sons, London, Vol. 1, 1930; Vol. 2, 1932).

[14] N.A. is a numerical designation of the wideness of cone angle for the largest light cone that may be transmitted by a given objective. Similarly, it refers to the proportions of a beam emerging from a condenser and is correlated with the size of the condenser stop. N.A. = (the sine of the half-angle—in air—of the marginal rays of the light cone) × (the refractive index of the medium into which the cone is projected). Cf. Fig. 6, *E*. The lower angle of the triangle is the half-angle of the light cone.

[15] For instance, in: C. R. Marshall and H. D. Griffith, *An Introduction to the Theory and Use of the Microscope* (G. Routledge and Sons, London, 1928), and Beck, reference 1. Shillaber, reference 1 shows photographs of starch grains with different illumination.

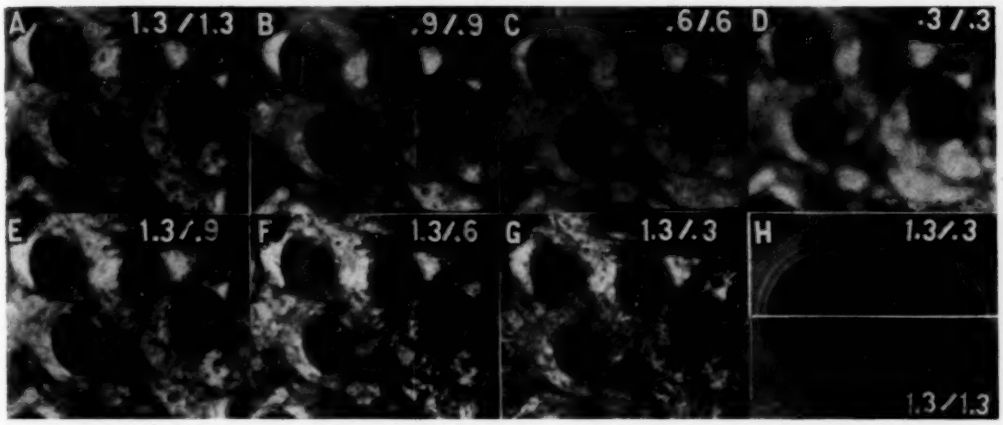

FIG. 1. *A–G:* Several cells of monkey parathyroid gland stained for mitochondrial granules in the cytoplasm; 3μ section ×1500. *A–D* shows the appearance with oil-immersion (*A*); and with 4-mm (*B*), 8-mm (*C*), and 16-mm (*D*) objectives, each with full cone illumination. The numerator and denominator represent, respectively, the N.A. of the objective and that of the condenser. *E–G* are oil-immersion figures but with reduced condenser apertures as indicated. *H* shows two globules of mercury photographed with reduced and with full cone illumination; the former illumination results in diffraction bands, ×1500.

shows a more common type of biological specimen (a stained preparation) photographed with different objectives, each one with full cone illumination (*A–D*), and also photographed (*E–G*) with one high power objective but with the condenser stopped more or less. The illustrations (*E–G*) show that smaller condenser stops result in increased depth of focus, heightened contrast, and decreased glare. A decrease in resolution also may be seen, and its extent may be compared with the lowered resolution (*B–D*) caused by objectives of lower aperture.

STANDARD METHODS OF LIGHTING

Nelson's method, which was particularly suited to high power work, was named "critical illumination." He placed an incandescent gas flame at some arbitrary distance from the microscope and adjusted it so that the flame could be viewed edge on. The essential features of Nelson's method were two: (1) the exact imaging of a luminous source in the field of view by raising or lowering the condenser level, and (2) the use of full cone, or nearly full cone, illumination. Exact focusing of a luminous source calls for a perfectly homogeneous luminous source, such as an incandescent flame, and since it is difficult to obtain large size sources that are equally luminous throughout, the method has been largely limited to the higher power types of microscopy. The flame, substage mirror, condenser, and microscope were aligned optically. An object was brought into focus in the field of view and, by raising or lowering the condenser, the flame was also accurately imaged in the field of view. Then the condenser iris was adjusted till the condenser aperture coincided with that of the objective (i.e., full cone illumination). This was done by removing the ocular and opening or closing the condenser iris till the uppermost lens of the objective, as seen through the open microscope tube, was flooded with light nearly to its margin.[16]

Because the filament of the electric lamp does not provide a suitably broad incandescent area, a filament electric lamp is a poor source for focused illumination. Ground glass or opalescent screens (Fig. 2, *B*) are grainy; they decrease the intensity of light noticeably; and, since diffusion is imperfect, the bright image of the filament shows through and produces a bright central spot and a non-homogeneous field. As a means of improving filament (and arc) sources, glass speculae (glass rods, perhaps a half-inch in diameter and

[16] The general method of examining the luminous appearance of an objective through the open draw tube (or by means of a suitable lens, through the ocular) was described by Abbe, reference 7.

several inches long) have been devised,[17] so that the light could be diffused by total internal reflection on the walls of the rod (Fig. 2, *C*). For high intensity illumination, light may be concentrated into a speculum by a lens or the speculum itself may be made with a trumpet shape.[18] The speculum, like the flame, though highly satisfactory for high power and immersion work, is a small-sized source and thus is useless for low power observation.

Light reflected from a suitable large white matte reflecting surface might be adequate for low power work, but such a system would generally be of low efficiency. An effective homogeneous illumination, however, is provided if light sources are enclosed as in a stereoptican projector or (integrating sphere) episcope (Fig. 2, *D*) having a white interior. A convex lens may or may not be used at the window of the episcope.

Auxiliary lenses and bull's eyes had been employed in microscopy since the seventeenth century and the condenser itself is nothing but a refined lens system for concentrating light. It is natural then that auxiliary lenses should have been tried in conjunction with the condenser to increase illumination intensity and the apparent size of a luminous source. When a fairly large collecting lens is aligned in front of a source of small area and at a suitable distance, an image of the source is projected through collecting lens and condenser to a focus at object level (Fig. 2, *E*). The bright image that appears in the field of view is noticeably larger than when the collecting lens is not employed. Because of this magnification, however, unusual care is required to assure a perfectly homogeneous source. Ground glass and diffusing screens before a light source are detrimental. In Fig. 2, *E*, one may see that the optical arrangement of the collecting lens and condenser is equivalent, though reversed, to the objective/ocular arrangement of the microscope proper.

This method of using an accessory lens was discussed[19] by Köhler (first paper), but Köhler's name is more importantly associated with a specific arrangement of the collecting lens in relation to the lighting beam. In this system, the light source, or its magnified image, is not projected to the field of view. Instead, the auxiliary collecting lens (or a pair of convex lenses) is so adjusted between the light source and the microscope that the image of the light source falls at the lower face of the condenser. When one looks into a microscope, so illuminated, the field is flooded with a limpid homogeneous illumination. The apparent source of illumination is the collecting lens itself (Fig. 2, *F*). A pencil or a finger placed in contact with the collecting lens surface will show up clearly in the field of view of the microscope. This is the optical system of the enlarging camera or projecting lantern and (apart from the substage mirror) differs only insofar as the condenser pro-

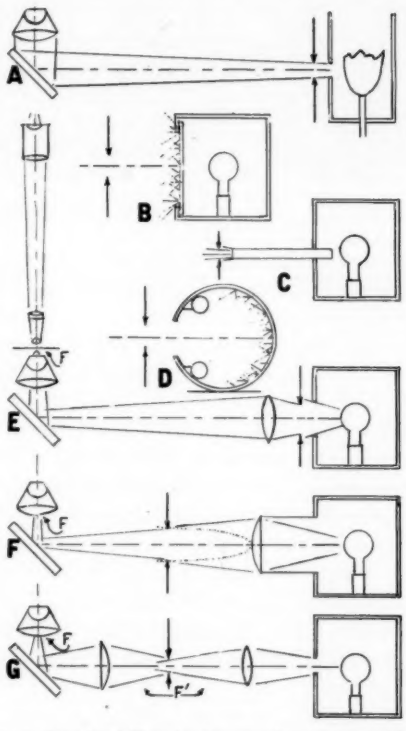

FIG. 2. Standard illuminating mechanisms. *A*, Nelson's method for critical illumination. *B*, ground-glass diffusing screen. *C*, speculum. *D*, episcope illumination. *E*, magnified source illumination. *F* and *G*, projection lantern type illumination. In the compound systems, *E*, *F*, and *G*, the level of the source image is shown at *F* and *F'*; arrows indicate the levels of field stops on which the condenser and microscope are focused.

[17] J. W. Gordon, J. Roy. Microscop. Soc., pp. 425–429 (1908). F. Welch, J. Roy. Microscop. Soc., pp. 34–37 (1930). R. E. Fitzpatrick, Stain Tech. **16**, 107–109 (1941).

[18] M. H. Knisely, Anat. Rec. **64**, 499–524 (1936).

[19] A. Köhler, Zeits. f. wiss. Mikrosk. **10**, 433–440 (1893). A. Köhler, "Mikrophotographie," *Handb. der biol. Arbeitsmethoden* (*Abderhalden*), Abt. 2, Part 2, No. 2, 1691–1978 (1931).

jects to a much closer focus than the long focus projecting lantern or the intermediately-focused enlarging camera. Slight irregularities of source homogeneity or ground glass close to the light source, as Köhler advised, in no way impair the homogeneous brightness of the bright field of the microscope. The method is especially suitable where high intensity illumination is required.

To adjust for the projecting lantern type of illumination, the image of the light source should be projected to the under surface of the condenser —in a darkened room, a clear image of the actual light source may be projected to a white card held against the lower face of the condenser—and the illumination may be tested by placing an opaque object before the collecting lens and adjusting the focus till the image in the field of view is clear. As in Nelson's method, the condenser is ordinarily adjusted for full cone illumination.

Though both of the compound illuminating mechanisms employing collecting lenses were apparently used prior to Köhler's first publication,[19] it was Köhler who explained the optical aspects of the methods and gave impetus to their adoption.

Actually, each change of objective or ocular in a microscope in some way alters its optical arrangement and hence its requirements for lighting. Accordingly, changes in an illuminating set-up are called for. These involve an adjustment of the N.A. of the condenser, altering of the distances between collecting lenses, or changes in the sizes and focal distances of the auxiliary lenses. Köhler designed a series of three lenses[20] which, singly or combined and arranged at appropriate distances (Fig. 2, *F* and *G*), filled the requirements for high, low, and intermediate powers of the microscope. The need for adjustments in changing from one lens arrangement to another and the requirements of space make the compound system somewhat clumsy and slow. Accordingly, they have been used chiefly in photomicrography. Lamps employing bull's eye lenses or flasks of water, as mentioned by Abbe,[4] have provided an approximation in microscopy. Too often, however, these are used without attention to focus level; for the projecting lantern arrangement, the level of focus of the source must be at the level of the lower face of the condenser.

Within the limitations indicated in the foregoing paragraphs, each of the methods of Fig. 2 is effective. Theoretical considerations have been presented[21] to suggest that direct imagery of a luminous source permits superior resolution. Photomicrographs with either illumination are similar, however.[22] Preference for one method or another is based on such factors of convenience as: whether high power or low power observations, or both, are to be made with the microscope, and such factors as the area of the source available, the working space available, the intensity of light required in relation to the source available, and the willingness to tolerate adjustments. There is as yet no inexpensive, compact illuminating device with means for varying intensity and suitable for the whole range of microscope powers.[23]

A point especially emphasized by Köhler relates to the importance of stops or diaphragms in the illuminating system. When a stop or diaphragm (field stop) is placed at an appropriate level (shown by arrows in Fig. 2) in the illuminating system, the image of the stop can be seen within the field of view of the microscope. For the projection lantern type of illumination (Fig. 2, *F* and *G*), the field stop (arrows) is placed in front of or between the collecting lens system; for direct source illumination using a collecting lens, it is placed behind the collecting lens and in front of the source (Fig. 2, *E*). In direct imagery of a source (Fig. 2, *A*, *B*, *C*, *D*), the diaphragm must be in front of the source. When the stop is suitably placed, the image of the field stop appears as a sharp-edged dark frame surrounding the bright field image. The size of the bright field thus is altered by constriction or dilation of the field stop opening. Appropriate adjustment of the size of the field stop will permit the bright field image bounded by the field stop image to coincide exactly with the field of view of the microscope.

The field stop is no mere adjunct to an

[20] Distributed for a time by the Zeiss Company. The more elaborate photomicrographic outfits are provided with lenses which can be combined to produce projecting lantern illumination.

[21] M. Berek, J. Roy. Microscop. Soc. **49**, 240–244 (1929).

[22] Allen, reference 1.

[23] The pancratic condenser of the Zeiss Company (advertised about 1937 *et seq.*) for illumination of objectives between N.A. 0.16 and N.A. 1.40 should meet illumination requirements most satisfactorily.

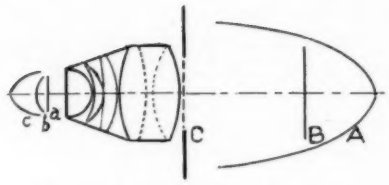

FIG. 3. Aspects of image formation by the condenser. To the right (the condenser field) and to the left (the region of focus), lines *A*, *B*, and *C* and *a*, *b*, *c* represent the location of conjugate points, *A*–*a*, etc.

illuminating system. Its importance is so great that it must be regarded as virtually the other half of the condenser.

When either the simple or compound method of illumination is employed using stops, full cone illumination is ordinarily indicated. The ocular is removed from the microscope, as mentioned in regard to Nelson's method, and the light-flooded area seen in the upper objective lens is adjusted to almost coincide with the aperture margin. A seven-eighths or nine-tenths aperture is a good practical approach to full cone illumination.

Köhler's contributions were significant, not in introducing methods of illumination, but in explaining them. The term "Köhler principle," which Köhler has disclaimed,[19] however, has come into use. This and the infelicitous "critical illumination," wherein "critical" does not have the usual implication in science of a transition from one state to another, have introduced confusion. Arbitrary distinctions have been made to differentiate two methods of illumination by some authors while others have used the terms indiscriminately, assuming that they apply to the same illumination. Because of the confusion, it would seem wise to scrap the terms and refer simply to controlled or regulated illumination. The essential features of "controlled" illumination would appear to be (1) full cone illumination by (2) a limpid homogeneous illumination, in (3) a system provided by stops, and (4) illuminating no more of an object than appears in the field of view of the microscope. The type of light source whether collecting lenses are or are not used, intensity, and other factors may be considered irrelevant to the term. When information concerning the mode of lighting is significant, it should be sufficient to refer to (1) direct source lighting (Fig. 2, *A*); (2) direct source lighting with speculum (Fig. 2, *C*); or with a collecting lens (Fig. 2, *E*); (3) episcope lighting (Fig. 2, *D*); or (4) projection lantern lighting (Fig. 2, *F* and *G*).

ILLUMINATION REQUIREMENTS OF THE MICROSCOPE

When the condenser is racked into a correct position in relation to the microscope, the foci of the two optical systems are so adjusted in tandem that one may look through the microscope and condenser, as through a telescope, and objects at a distance may be seen enlarged in the field of view. Within a narrow range of adjustment, the condenser may be racked toward or away from the objective, and far or near objects, respectively, are brought into focus. The region of clear focus seen in the field beyond the condenser, as viewed through the microscope, is markedly concave. The surface of curvature is that of a paraboloid. This curvature of condenser field is a characteristic image defect of condenser optics and it is illustrated in Fig. 3. Lines *A* and *a*, in the figure, represent the location of conjugate points. If the condenser is racked into proper position, both the image of the concave condenser field and of the object as well are simultaneously viewed through the microscope. On the other hand, the plane surfaces in the field beyond the condenser (*B* and *C*, the latter considered as the plane of the condenser stop) are imaged at curved surfaces (*b* and *c*). Through a microscope focused on a plane object, such images appear largely out of focus. Were it not for this effect, a ruler, or gridlines on a transparent plate, placed at suitable level in the condenser field could be used for measurement, or for counting, of objects within the field of view.

One other level within the microscope that coincides optically with the two mentioned, concave condenser field and object plane, is the plane of the ocular diaphragm where ocular grids and pointers are placed. The three planes of focus (*A*, *B*, and *C* of Fig. 4) are simultaneously projected to the retina (*D*) or camera ground glass. No other levels are clearly focused.[24] If diaphragms or stops are placed at any of the three levels of focus, the bright image of the illuminant,

[24] A. E. Wright, *Principles of Microscopy, being a Handbook to the Microscope* (The Macmillan Company, New York, 1917).

as transmitted to the eye or camera, is constricted and the image is bounded by the darkened margin of the image of the constricting diaphragm. Similarly, if there are diaphragms at each level, that diaphragm which is most constricted in relation to the optical beam provides, through its image, a margin to the field of view. Ordinarily, the image of the ocular diaphragm forms a sharp limit to the field and determines the width of the effective cone of light entering the eye, but the field may be further reduced by tiny stops at object level—an impractical method—or by larger ones encircling the concave field of condenser focus. Field stops as used by Köhler (Fig. 2) do this.

Because of the concavity of the condenser field, the margin of the field stop and the light source, or collecting lens, beyond may be simultaneously brought into view. Conjecturally, a luminous source ideally designed for the microscope should present a concave paraboloid surface to the condenser, but practically this is unimportant. In dealing with the imagery of large areas of perfectly homogeneous luminosity, i.e., where point-to-point brightness differences over a surface may be ignored, the whole concept of focus becomes meaningless. One cannot focus a camera at a homogeneous clear blue sky! Thus, when a condenser is so adjusted that the image of a field stop appears in the field of view, good lighting may result even though the light source is well beyond the level at which sharp imagery would be possible. Light from such a source traverses every point of the curved condenser field in passing to the condenser and the field of view presents a limpid luminosity. In other words, if the field stop is accurately imaged, the luminous source or its apparent source (collecting lens), supposing it to be broad and homogeneous may be presented as an "out-of-focus" image. Moreover, where scratches or dust are present, this is preferable to direct imagery of a collecting lens.

Any of the lighting mechanisms of Fig. 2 will serve if the light meets the demands of homogeneity, size, and brightness. Obviously, filters, neutral wedges, cooling cells, etc. should not be placed near the region of the curved field of condenser focus. With the projection lantern type of lighting, filters or other transparent media may be placed near the condenser—but not near the collecting lens—without scratches or dirt being too objectionable. It is best, however, to keep the space between the condenser and its field of focus free from possible obstructions. It is probably better to place filters, etc. between the light source and the collecting lens.

A dusty substage mirror, since it is so near the condenser, for instance, may not modify the light of the bright field image significantly. The field stop margin, however, will not show clearly. Double images and aberration of the field stop margin result characteristically from ordinary,

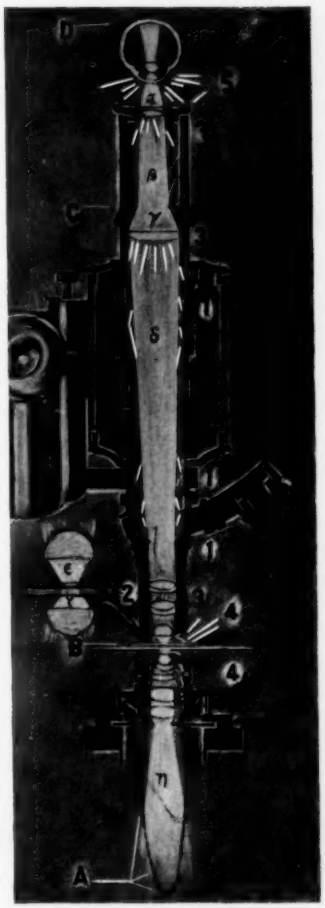

FIG. 4. Diagram of the optical beam through the microscope and eye. Greek letters refer to segments of the beam; *A*, *B*, *C*, and *D* are levels of simultaneous focus; numbers refer to sites of glare production.

substage, rear-surface mirrors. A substage prism or a front surface mirror avoids double images of the field stop.

THE EFFECTIVE BEAM

The basic requirements for microscope illumination, apart from homogeneity and brightness, may be determined by retracing the illuminant beam through the microscope and condenser. In Fig. 4, the effective beam is seen to be subdivided into a series of truncated cones (Greek letters). These lie either between planes of focus (*A*, *B*, *C*, and *D*) and the adjacent lenses or in interlenticular spaces where no planes of focus intervene. The proportions of these zones are rigidly determined by the optical constants of the lenses employed and are quite different when different objectives or different oculars are substituted (Fig. 5). It will be apparent, for instance, that in Fig. 4 longer or shorter oculars will change the length of cone δ; changes in tube length will do the same thing. It is particularly to be noted that the shape of cone η, below the condenser, is a function of the optical system of the microscope and that different oculars, objectives or tube length alter the light requirements.

Authors in the past have routinely assumed the fiction that parallel rays are required for condenser illumination. By looking through a microscope and condenser system toward a large bright surface, one may measure widths across cone η for various ocular and objective combinations, and the proportions may be plotted (Fig. 5).[25] The figure shows that an approximately parallel beam was required for the 4-mm-10x combination, but that converging or diverging beams were called for by the other combinations. In general, higher objectives and higher oculars require a narrower cone of illumination than lower powers do. The concavity of the condenser field, accordingly, is subtended at its depth and the amount of curvature appears small.

Field stops have as their function the limiting of the actual lighting beam to the size of the effective retraceable beam. The latter may be defined as the maximal beam from an illuminant that, apart from lenticular reflections, is entirely refracted by the optical system of the condenser and microscope and is projected to the eye or camera. Light of a wider beam on entering the microscope falls external to the effective beam and is reflected on the lens mountings and draw tube to form a source of glare. The essential feature of controlled illumination is that it automatically provides both a homogeneous full cone illumination and a minimum of glare. In practice, the actual optical beam is adjusted to the proportions of the effective beam by the field and condenser stops.

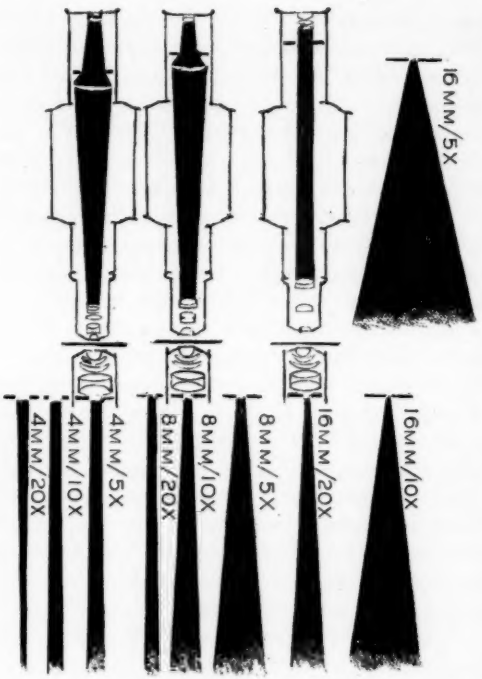

FIG. 5. To satisfy completely microscope requirements for the various objective-ocular combinations, beams must have the proportions shown below and to the right.

ILLUMINATION FROM THE CONDENSER

Figure 6 depicts the shape of illuminating beams emerging from the condenser for different adjustments of the condenser and field stops. It is this beam that actually illuminates a specimen; therefore, a concrete visualization of the shape of the beam and of its modification by

[25] A Spencer and a Leitz microscope, provided with apochromatic objectives, compensating type oculars, and corrected condensers, formed the basis for the various observations made. Occasionally, points were checked on student type microscopes of the same and other makers.

stops is basic to an understanding of practical illumination.[26] In the illustrations, the light beam has the shape of a double cone consisting of a lower and an upper segment jointed at a constricted waist. At *A*, both the condenser iris and the field stop are open rather widely; the segments have markedly converging or diverging sides, and the plane of condenser focus, at the waist of the double cone, is of large diameter. In contrast, as at *C*, the double cone is thin and the region of focus (at *x*) is much constricted when the two stops are made small. At *B*, the very broad double cone of light is produced when the condenser, with wide open iris, is directed toward a broad illuminant such as the sky. The circles at the plane of focus (at *a*) represent the relative sizes of the fields of view of three objectives (4 mm, small circle; 8 mm, intermediate circle; and 16 mm, large circle—all with the same ocular). The wasted and glare-producing light peripheral to any of the fields of view is considerable.

Sketches *D*, *E*, and *F* show the shape of double cones that are adjusted for controlled (full cone) illumination appropriate for the 4-mm, 8-mm, and 16-mm objectives (with 10x ocular). The diameters of the constricted waists of the double cones are equal to the diameters of the fields of view of the various objectives and are dependent on the degree of constriction of the field stop. The slant of the marginal rays varies from 40–45° (*D*) to a much smaller angle and depends on the degree of constriction of the condenser diaphragm.

[26] The shape of the condenser beam is visualized in a dark room if a vertically held cigarette paper or a lightly exposed photographic film is moved back and forth through the beam. The proportions of the beam may be determined also by placing paper or a film horizontally on the microscope stage and racking the condenser up and down; the condenser beam is intercepted and a circle of measurable diameter may be noted for each level of the condenser. The circles are measured while they are being viewed by the microscope provided with lowest power lenses. An inch cube of uranium glass on the stage and in a darkened room shows a similar double cone from the condenser, but such a cone is a little less wide angled than one projected into air. Cf. Marshall and Griffith, reference 15; Muñoz and Charipper, reference 1. The cone within a slide in oil-immersion work is visualized this way.

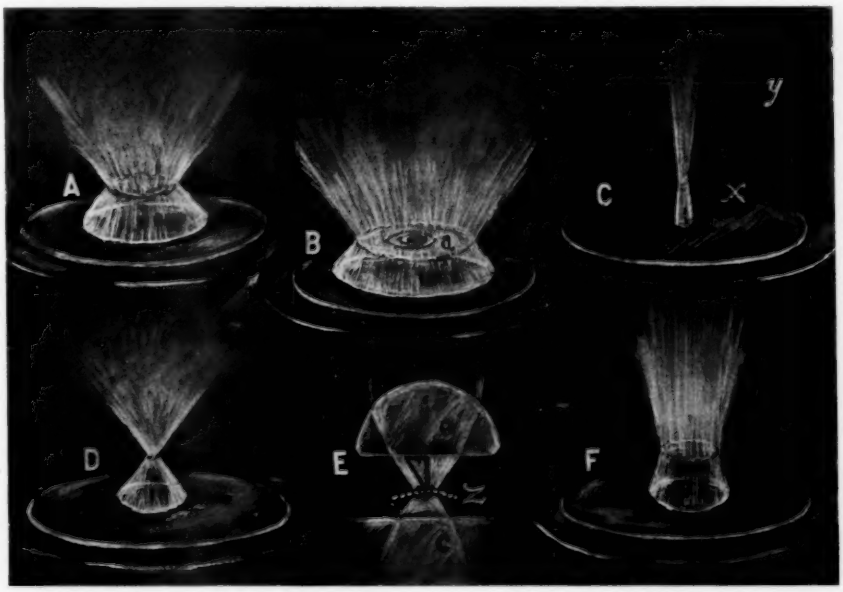

FIG. 6. The shapes of beams emerging from the condenser; *A*, with wide condenser and field stops; *B*, with wide open condenser stop and no field stop; *C*, with narrow condenser and field stops (*x* is the level of the image of the field stop and *y* that of the condenser stop); *D*, condenser and field stops regulated for controlled illumination of the 4-mm objective; *E*, same for the 8-mm objective (*o* = objective, *c* = condenser, *z* = level at which the field stop image appears; the lower angle of the triangle is the half-angle of the beam and represents the angular aperture of the condenser beam); *F*, controlled illumination for the 16-mm objective.

Much wider or more slanting cones than those figured are produced when a condenser beam is directed into immersion oil of the same refractive index as the condenser glass. With wide open condenser aperture, beam angles of 130° to 140° may be possible.

Such cones as those in the lower sketches of Fig. 6 are adjusted to the requirements of objectives while those shown above are not. The field stop determines the width of the waist of the double cone while the iris below the condenser determines the slant of the sides of the double cone. When either objectives or oculars are changed, the width of the waist of the double cone must be changed at the field diaphragm or the area of the field of view will not be illuminated exactly. Changes of the slope of the sides of the double cone are required for controlled illumination when objectives are changed, but no change of slope is required for changed oculars.

At the waist of the double cone is the image of the field stop (x, Fig. 6, C; z, Fig. 6, E) while at some other level, as at y, there is an image of the condenser iris. As shown in Fig. 3, the level of the image of the condenser stop will appear at some level above the condenser face transecting line c. Image y, which is several millimeters above the condenser lens surface for small condenser apertures and very close to the condenser lens for large apertures, is entirely external to the double cone, i.e., in the dark region external to the field of view, when controlled illumination is employed. The image of the condenser iris, however, may be seen through the use of a low power objective with a very wide field of view if one focuses at a suitable level (several millimeters) above the object plane (i.e., waist of the double cone). Obviously, the image of the condenser iris in no way impinges on, constricts, or controls directly the proportions of the double-cone illuminant beam. The iris of the condenser may, of course, be so constricted that its image actually impinges on and apparently constricts the diverging cone (Fig. 6, C), but such a beam is no longer controlled (i.e., full cone).

When no field stop is provided and a broad source of light is used—the sky, for instance—a new type of double cone with the image of the condenser iris at its waist is formed (Fig. 6, B). But, differing from the double cone of controlled illumination, this type of double cone is altered in three ways by adjustments of the condenser iris. Constriction of the condenser iris simultaneously raises the level of the waist of the double cone in relation to the level of the condenser face, decreases the width of the waist, and decreases the slope of the diverging cone. Such a beam will not fill the aperture of the objective and at the same time provide an illuminated area equal to the field of view. This type of beam cannot be regulated to illuminate the field completely and at the same time eliminate tube glare. Such a beam is not recommended; the use of a field stop in addition to the condenser stop alone permits adequate control.

ADJUSTMENTS FOR CONTROLLED ILLUMINATION

Basically, controlled illumination is lighting by a cone of rays (η in Fig. 4), the proportions of which are regulated by a stop at the illuminant and by the iris diaphragm of the condenser. Through the stops, the cone is so adjusted that the actual illumination provides an effective beam (as defined). The aperture of the objective is completely used and none of the refracted light falls on lens mountings or draw tube to form glare. The standard adjustments for obtaining controlled illumination are dealt with by Nelson[11] and Köhler,[19] and summaries are to be found in texts on microscopy and photomicrography. Briefly, though with slight differences from the original method, the procedures are:

1. Center the condenser in relation to the microscope. Optical alignment is a necessary condition for avoiding illumination of graded intensity across a field and uneven specimen contrast.

2. Place the illuminant, provided with an adjustable (diaphragm) field stop or series of stops, at a convenient distance in front of the microscope (Fig. 2). Henceforth, use the same stop-to-microscope distance. A rigid platform with fixed places for microscope and illuminant, and its stop, is advisable. Then, stop diameters are constant and calibration of a field stop becomes feasible.

3. Focus the microscope on a specimen on a slide using the objective and ocular intended for observation.

4. Rack the condenser till the image of the field stop is in focus in the field of view of the object. Throughout, use only the plane surface of the substage mirror. A plane front surface mirror or a prism with silvered oblique face is recommended.

5. The illuminant used will be one of those illustrated in Fig. 2. For one of the simple illuminants, the actual luminous source is moved forward or back so that it appears in focus within the field of view—or it may be moved away till the source image is actually out of focus. The same applies to compound illumination involving imaging of the source. When the projection lantern type of illuminant is employed, check to see that the image of the light source falls at the lower face of the condenser.

6. Remove the ocular and look into the draw tube at the rear lens of the objective; open or constrict the iris diaphragm of the condenser so that the edge of its image as seen on the rear lens of the objective is just visible. This adjustment provides full cone illumination. A nine-tenths cone is often preferred to full cone illumination since glare is less.

7. Replace the ocular.

8. Adjust the size of the field stop so that the image of its margin coincides with that of the ocular diaphragm.

Procedures 3, 4, 6, 7, and 8 are ordinarily required as a routine whenever a different objective or ocular is employed, since the adjustments for controlled illumination of one optical system do not apply to another. The other procedures (1, 2, 5) are usually required only on beginning work. Some simplification of the routine is possible if tube length, the type of illuminant, and the field stop-to-microscope distance are rigidly standardized. Then, through procedure 6, the position of the condenser-stop lever is determined for each objective in use and a scale indicating lever position—actually, a scale of condenser N.A.—may be marked on the condenser housing. Now procedures 6 and 7 may be deleted and instead the condenser lever is merely set at a (N.A.) position corresponding with the objective N.A. A further convenience of this arrangement is that the condenser aperture may be reduced by definite steps to produce cones of specified size.

The procedures outlined automatically insure that the actual illumination corresponds with the effective beam required and extraneous glare-producing light is in large part eliminated. If the procedures indicated are standardized, preliminary checking of steps 1, 2, and 5 is required at the beginning of work; 3 and 8 together with the adjustment of the condenser aperture are called for at each change of either objectives or oculars. The latter procedures become routine easily.

MICROSCOPE GLARE

Over a period of years, the optics of the microscope have been corrected to such an extent that the principal remaining defect is caused not by optical aberrations but by glare. Hazy or milky microscopical images prevent full realization of the powers of the microscope since visual contrast is so diminished that high visual resolution becomes impossible. Moreover, glare introduces a serious source of eye fatigue if observations are at all prolonged.[27] In photomicrography, too, hazy images prevent the attainment of crisp detail and contrast in negatives.

The problem of microscope glare has been recognized in some of the texts on microscopy—Shillaber, Belling, Beck[1]—but the most pertinent observations have been those by Beck and by Fabry.[28]

Though glare is in part an inherent defect of the microscope (and eye), some of the causes result from the way a microscope is used. Thus, with proper care, some of the glare may be avoided or reduced. Unwanted light may get into the optical system in several ways to produce a cumulative effect. Obviously then, the reduction or elimination of glare at one place and another will progressively improve visual contrast. Glare may be considered from its sites of origin as: (1) tube glare, (2) glare from lens mountings, (3) lenticular glare, (4) slide glare, and (5) eye glare. These five sites are indicated by numbers in Fig. 4, and the types of glare are discussed serially below.

[27] The effect of loss of contrast on visual resolution and the significance of other factors on visual performance in microscopy have been dealt with in a separate paper. J. Opt. Soc. Am. **34**, 711 (1944).

[28] C. Beck, J. Roy. Microscop. Soc., pp. 399–405 (1922). C. Beck, J. Roy. Microscop. Soc., pp. 1–8 (1933). C. Fabry, Proc. Phys. Soc. London **48**, 747–762 (1936).

Tube Glare

This type of glare is caused by reflections on the walls of the draw tube or body tube of the microscope. It appears either when an area at object level greater than the field of view is illuminated by the condenser or when the cone of condenser illumination is wider than the full cone required by an objective. In either case, a beam wider than that required to subtend the field lens of the ocular is produced within the body tube of the microscope (cone δ, Fig. 4), and the more peripheral rays fall on the walls of the draw tube or body tube to be reflected into the ocular as non-image forming and glare-producing light. This type of glare may be accentuated for illustration by placing a shiny brass tube into the microscope between the objective and ocular; a brassy colored haze caused by reflections suffuses the field peripherally. The wide body tube of research microscopes, together with the draw tube, forms a light trap (Fig. 4) that eliminates the wider rays emerging from the objective and the stop at the lower end of the draw tube is of some importance. Restriction of an illuminant beam by a field stop as required by controlled illumination is, however, the really significant method of avoiding tube glare.

Glare from Lens Mountings

It is probably somewhat arbitrary to separate this type of glare from tube glare since it likewise is caused by light that falls peripheral to some of the lenses of the catenation in the optical system of the microscope. Separate analysis of glare produced at lens mountings, however, should lead to an understanding of both types of glare. The principle of glare production may be considered in relation to the hemispheric first lens of an objective, Fig. 7, *A*. Rays from the field of view circle at object level (*A*), pass into the hemispheric lens and are refracted toward and through the series of lenses above to produce an image. Simultaneously, however, bright points peripheral to the field of view (*B* and *C*) are refracted to a region peripheral to the border of the second lens of the objective. Such light falls on the lens mountings and is in part reflected into the second lens as non-image forming but glare producing light.

Oblique rays of light may enter the objective mainly as reflections from the surface of a cover glass and slide (Fig. 4). Such glare may be accentuated for illustration by placing on the stage of the microscope a shiny plate of brass bored with a hole slightly larger than the field of view of the objective used. The plate is illuminated from above by oblique light, and the hole is placed so that its margin is outside the field of view. A characteristic brassy-colored haze suffuses the field chiefly at its periphery. Similar reflections from a slide surface may be recognized if

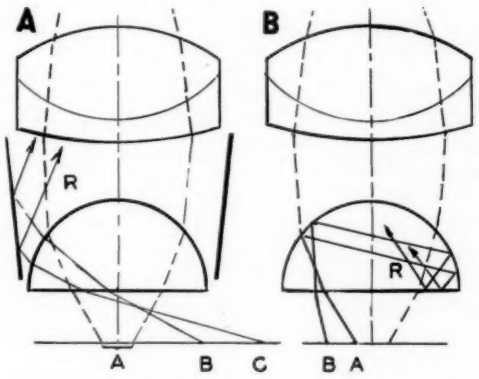

FIG. 7. *A* shows the refraction of rays *A*, *B*, and *C* through the first lens of an objective; rays *B* and *C* outside the field of view of the objective (zone *A*) are reflected (*R*) on lens mountings as glare producing light. *B* shows two rays (*A* and *B*) reflected (internal reflection) within the front lens of an objective and forming a source of glare (*R*).

the oblique light is colored differently from that through the condenser. These reflections, of course, are absent with oil immersion or if the microscope stage is shielded from extraneous light.

A suitable test for the detection of glare producing light at one's work table is provided by tiny globules of mercury on a slide and in the field of view. When the stage is shielded, the globules are uniformly dark; but, when extraneous light is permitted, the extent of reflection at the surface of a globule is an index of the glare caused by reflections. For photomicrographic work, Shillaber suggests placing a collar around the objective to avoid this type of glare.

Lenticular Glare

This third type of glare is inherent in the optical system of the microscope. Light of the optical beam within the microscope is partly reflected (4.5+percent) at each air-glass interface through which the beam passes and at interfaces between glasses of differing refractive index. Ten glass-air interfaces are shown in Fig. 4, and in a binocular instrument with prisms there are many more. If as much as 5 percent of the light is reflected at each of ten glass-air interfaces, the cumulative loss of image forming light from the optical beam of a microscope amounts to about 40 percent. Light reflected from lenses is in part re-reflected into the optical system as stray light. Fogging by such reflections is accentuated in the higher power systems where, because of more lenses, the cumulative effect is greater. If the recently developed methods for producing anti-reflecting coatings for lenses[29] prove practicable for the microscope, some of the lenticular glare may be reduced by using coated lenses with light of suitable color. Such coatings, however, though reducing reflections at the first face of a lens do not reduce reflections at the second or glass-to-air interface.

Some scattering of light[30] occurs at interfaces between media of different refractive index (*viz.*, glass-air) so that an actual surface—no matter how clean—may show a slight luminosity; this effect is called flare. Scattering of light is seriously enhanced if lens surfaces are dusty or greasy. Accordingly, exacting attention to keeping lenses free of dirt is the first contribution of the microscope user toward the reduction of glare.

The amount of reflection increases as the angle of incidence of light pencils, in relation to the glass surface, deviates from the normal. Since first (air-glass) surfaces of microscope lenses are ordinarily convex or plane (Figs. 4 and 5), light is reflected peripheral to the optic beam passing through the microscope and does not contribute to glare. At the upper or second surfaces, however, reflections are directed axially to be re-reflected within a lens as glare. This is illustrated especially by the lowermost objective lens in which the convexity may equal or exceed a hemisphere (Fig. 7, *B*). As diagrammed, light that is widely incident to the curved surface of such a lens is reflected in considerable quantity, and, on re-reflection, it provides a basis for a suffusing haze. If incidence angles at the curved surface exceed the critical angle for total internal reflection (roughly 40–45°), glare will be extensive. Designing by manufacturers of the shape and proportions of lenses obviates glare caused by total internal reflection when the lenses are used properly, but partial reflections persist. Appreciation of lenticular glare by the microscope user will allow him to reduce the effect to a minimum. The adjustment of proper illuminating cones (i.e., controlled illumination, Fig. 6, *D*, *E*, and *F*) is a working essential. The periphery of the field of view is more likely to be a source of glare than the more central region. By constriction of the field stop well below the requirements of controlled illumination, the bright-field image of a microscope may be reduced to a small portion of the field of view; this cuts out the portion of the field that is most apt to produce glare, yet full aperture illumination is still employed. Constriction of the field stop, thus, becomes a practical measure; in photomicrography, it is always advisable to reduce the field stop so that the image is no larger than the film area; in microscopy, fine or crucial observations should, at least, be checked with the reduced-field full cone illumination.

Slide Glare

This fourth and serious type of glare arises when a microscope slide is added to the optical system of the microscope and condenser. Aspects of the optical system that is attained when a slide is added to the microscope system and the factors relating to slide glare are treated in relation to Figs. 8 and 9.

When a slide and cover glass are placed across a double-cone condenser beam, the beam is deformed (Fig. 8, *A*) by the refractive properties of the glass and the actual angle of rays within the glass is decreased as compared to the angle in air. On emerging, however, the beam again acquires the direction it had on leaving the condenser lens.

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[29] K. B. Blodgett, Phys. Rev. **55**, 391–404 (1939). C. H. Cartwright, Phys. Rev. **57**, 1060 (1940). F. L. Jones and H. J. Homer, J. Opt. Soc. Am. **31**, 34–37 (1941).

[30] R. W. Wood, *Physical Optics* (The Macmillan Company, New York, 1934), third edition.

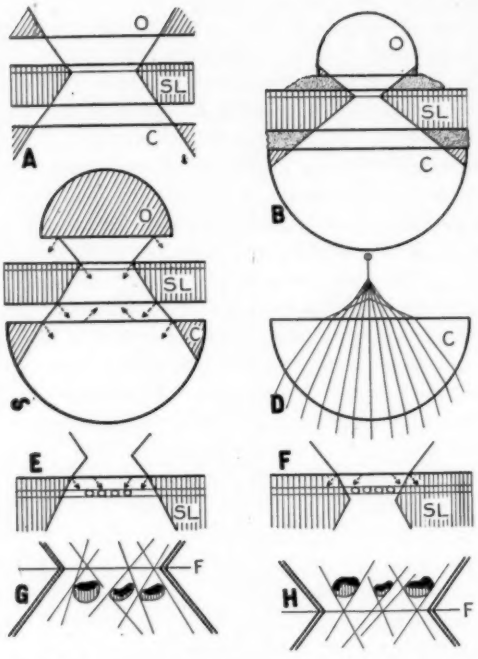

FIG. 8. *A*, ray tracing from condenser (*C*) through slide (*SL*) and into objective (*O*). *B*, same but with oil-immersion (oil-stippled). *C*, arrows represent external and internal reflections at the several surfaces. *D*, spherical aberration in a beam emerging from an unimmersed condenser top lens. *E*, centrally directed reflections with a high condenser cone. *F*, peripherally directed reflections with a low condenser cone. *G*, objects on a slide below the level of focus (*F*) of a high condenser cone. *H*, increased contrast of particles caused by less top illumination when the level of focus (*F*) is below the objects.

Those rays that are reflected at glass-air interfaces thus have a smaller incidence-reflection angle for internal reflection than for external reflection.

With oil immersion (Fig. 8, *B*)—oil between the condenser and slide and between the slide and objective—there is no deviation of rays caused by differences in refractive index and the beam may be adjusted to angles greater than the critical angle for internal reflection (i.e., numerical apertures greater than 1.0 are possible). Reflections that occur ordinarily at glass-air interfaces (Fig. 8, *C*) and flare are eliminated when oil-immersion is used.

Image defects such as spherical aberration, chromatic aberration, coma, etc. have been largely eliminated by microscope designers, and significant residual aberrations are associated only with the condenser. Such aberrations of the condenser (Fig. 8, *D*), however, are practically of minor significance; they affect only the image of the field stop. An appreciable amount of spherical aberration, on the other hand, may be introduced into the optical system of the microscope when a microscope slide is placed across the condenser beam. Adjustment of the correction collar of high power objectives for cover-glass thickness is an important compensation. It is only with oil-immersion, though, where the whole space between objective and condenser is of the same refractive index that slide aberrations, along with slide reflections, are entirely eliminated.

Figure 9, *A* is a conventional illustration of spherical aberration; because of aberration, the image of a point becomes a diffuse zone $(x-y)$ instead of a discrete point. Similarly, a converging or diverging beam (Fig. 9, *B* and *C*) passing through a plate of glass with plane surfaces shows spherical aberration. A slide in a condenser beam thus provides conditions that result to some degree in image aberration; the wider the condenser cone, the greater the aberration appears. When rays conforming to an image pattern are refracted through surfaces that produce spherical aberration, a dark area in an image (*s* of Fig. 9, *D*) is not crisp edged and the area of focus is a blur spot suffused with a glare-like haze (*s'*).[31] Light radiating from details of an object on a slide and refracted at a cover-glass surface will be contaminated somewhat, as compared with the oil-immersion system, by haziness owing to spherical aberration.

When unimmersed, the undersurface of a slide gives rise to reflections (Fig. 8, *C*) that are an objectionable source of glare. Abbe[4] and others (Belling[1]) have indicated that oil, or water which is less messy, may, with advantage, be placed between a slide and condenser to eliminate or reduce reflections even when dry objectives are used. Slides with specially prepared antireflecting

[31] The similarity of spherical aberration and glare in obscuring an image may be illustrated by looking through a microscope and condenser so adjusted that objects at a distance (i.e., out of a window) are seen as through a telescope. The degree of image fuzziness is largely owing to residual aberrations of the condenser. Opening the condenser increases spherical aberration and an increased haziness appears.

coatings[32] reduce glare perceptibly though they are not of as much value as condenser immersion, using oil.

Glare at unimmersed cover-glass surfaces owing to internal reflection (Fig. 8, *C*) may be reduced to a large extent by careful adjustment of the level of condenser focus. When the condenser is racked up so that the converging beam reflects (by internal reflection) on the cover-glass surface, light is reflected centrally (Fig. 8, *E*) and a bright central spot or a glaring annulus partly obscures the field of view. The object, in effect, is illuminated both by transmitted light and by direct light. When the level of the condenser focus is slightly below the cover-glass surface (Fig. 8, *F*), reflections are thrown peripherally and do not obscure the field.

Shillaber indicates a special source of glare found chiefly in the examination of crystals. Here, reflection from one crystal to another neighboring crystal may cause a mixture of transmitted and reflected light as well as glare. In biological work, the glare effects of light scattered in a tissue are less serious.

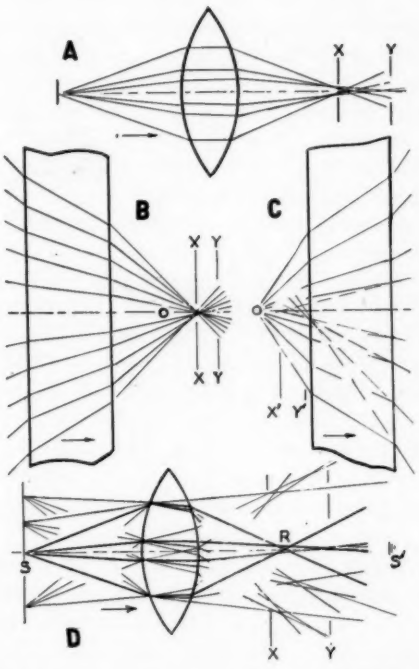

FIG. 9. Ray tracing through a lens showing spherical aberration. Focus level is over the span *X*–*Y*. *B*, rays converging to a focus at *O* are distorted by spherical aberration when a flat plate is interposed and the focus spreads between *X* and *Y*. *C*, rays diverging from a point (*O*) are distorted in traversing a plate so that the projections of the emerging rays intersect between *X* and *Y*. *D*, spherical aberration of a lens with a zone of focus at *X*–*Y*. Point *S* is projected to level *R* as a blur circle of the size shown at *S'*.

[32] Several slides, coated by the Cartwright method (reference 29), were prepared for me through the kindness of Professor R. C. Williams. When light of an appropriate color was used, a slight but perceptible decrease in glare was evident. Since coated slides do not alter the amount of internal reflection, the major problem remains.

Eye Glare

The last type of glare to be mentioned arises external to the optical system of the microscope and in a strict sense is limited to visual work with the microscope. Glare may be added to the visual image by extraneous light entering the eye from the side (i.e., from the periphery of the field of vision) or by reflections from the top of the ocular or from spectacle lenses. This type of glare is in part analogous to the fogging defect in photomicrography caused by cracks in a camera bellows. In addition, however, spot glare sources have a pernicious effect on visual efficiency because of peculiarities of the psychophysiology of the eye. Such glare affecting the eyes may, of course, be eliminated by the use of an eyeshade, but, it should be noted, total exclusion of peripheral light is not ideal since dark surroundings provide conditions incompatible with superior visual acuity. This problem is the subject of a further paper.[27]

OBJECT CONTRAST

Minute solid particles with reflecting surfaces —fine pumice, scrapings of dried India ink, or tiny globules of mercury—if placed on a slide in a widely converging beam, are illuminated on their sides and, to some extent, on their upper surfaces (Fig. 8, *G*). Their profiles as seen in the microscope are not clear. In a diverging beam, on the other hand (Fig. 8, *H*), the upper surface is less illuminated and the particles show as if in silhouette. With light absorbing particles (stained specimens), the silhouette effect is likewise accentuated when the level of the condenser focus is just below the object level. The adjustment is desirable with both dry and oil immersion objectives. Beck[1] indicates that more lines per inch

are resolved in test gratings if the condenser focus is slightly below the grating level than if the focus level coincides with or is above the grating level.

Constriction of the condenser iris decreases the obliquity of the illuminating rays emerging from the condenser, and the microscope image shows increasing contrast of edges and a decrease in haze. The upper lens of an objective, however, as seen through the draw tube, shows a dark periphery and indicates a decrease in the working N.A. of the objective and a resultant decrease in the conditions for high resolution. Decrease in beam obliquity below the aperture requirements of an objective may be expected to decrease the glare-like effects of spherical aberration, reduce the amount of lenticular glare, and reduce the possibility of side and top illumination of objects in the field of view. Both reflecting and light absorbing objects show increasingly dark contours. With refractile objects like diatoms, however, the nature of the surface configuration and the refractive index of the mounting medium alter the distribution of the emergent light and the resultant microscopical appearance changes as the obliquity of the light is changed. Even here, if the medium is of suitable refractive index, the general contrast is enhanced by light of less obliquity than required to fill the objective.

In general, contrast appears to be highest and glare the lowest when the condenser iris is constricted to an N.A. value of about a third that of the objective used. When the condenser is reduced in this way, a beam roughly like that of Fig. 6, *C*, or a beam intermediate to those shown at *C* and *F* is produced. With such a beam, the central rather than the peripheral parts of the objective are used in image production. Images of both the field diaphragm and condenser diaphragm impinge on the beam (Fig. 6, *C*) and the diverging cone so limited is in a sense projected through the objective lenses with a minimum of stray light. This modification of controlled illumination may be called contrast illumination.

Further constriction of the condenser iris gives rise to diffraction effects—alternate light and dark bands bounding the contours of objects —such as those illustrated in Fig. 1, *H* and *G*. As the iris is opened, the contour bands fade and maximum edge sharpness and contrast appear greatest when the dark bands have just faded.

In practical microscopy, the condenser iris should be toyed with constantly so that contrast and controlled or intermediate degrees of illumination are employed in varying sequence. Some method of increasing and decreasing illuminant brightness apart from the condenser should be available, too. Through condenser adjustments, the observer provides his eye with fresh visual impressions of an object. Apart from brightness differences, variations in the condenser iris produce differences in resolving power, in object contrast, and in the depth of focus perceivable by a given objective (Fig. 1). In photomicrography, a desirable compromise involving these factors is to be sought by condenser adjustment. Though a three-quarters to nine-tenths aperture is a common compromise, it is best in practice to regulate the condenser till the exact appearance sought (Fig. 1) shows in the ground glass.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 34, NUMBER 12 DECEMBER, 1944

Visual Factors in Microscopy

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(Received September 20, 1944)

In work with the microscope, maximum returns in the way of crisp imagery, in detail and in visual comfort, as well, are obtained only when the conditions of illumination are carefully adjusted. Since the eye is a physiological mechanism, special conditions beyond those basic to good photomicrography are necessary for high visual efficiency. The problem of vision through a microscope is discussed and the literature relative to visual acuity is reviewed. Conditions for high visual acuity and visual comfort are outlined.

THERE are probably few fields requiring acute visual performance in which the eye is routinely subjected to more insults than in microscopy. This is particularly true in student laboratories in biology where nondescript conditions of lighting are the rule. The same standards are generally carried into advanced and research work with the microscope. Eyes are forced to adjust for poor light, and both visual discomfort and indifferent work must be expected.

The problem undertaken here is the analyzing of aspects of the visual task involved in microscopy and the determining, insofar as possible, of the conditions that obviate visual discomfort and which promote the highest visual efficiency. A rough or generalized knowledge of the eye will be assumed, but special points that are pertinent will be mentioned as a setting for the later discussion.

STRUCTURAL AND DIOPTRIC ASPECTS OF THE EYE

The eye, in part, is an optical instrument akin to the camera obscura. Effective light stimuli, if unobstructed, enter the eye through a visual angle of about 160°. The nose, eyebrow, cheek, lids, and the outer orbital margin, however, obtrude into the periphery of the visual field on one side or another depending on how the eye is directed and practically the field of view is somewhat constricted below 160°. The eye, when centered over the microscope (Fig. 1), is presented with a field of view divided in three concentric areas: Centrally, the bright field of the microscope subtends a solid angle of roughly 25–30°, peripheral to this and subtending the rim of the ocular is an annulus of nearly twice the bright field angle, and still more peripherally is the general surround within the field of vision. Light characteristic of each region simultaneously enters the eye. As the gaze shifts over the bright field, the eye rotates in its socket and the whole projected image—all three zones of the visual field—at the rear of the eyeball move like the image on a camera ground glass when the camera is moved. A whole image stimulus, thus, is not only distributed over the whole retina but is constantly shifting topographically in relation to the retinal surface.[1]

Unlike the photographic plate, however, the retina is not equally sensitive throughout. Østerberg's careful study[2] (and that of Polyak[3] also) has shown that the density per unit area of the percipient or light sensitive elements (cones and rods) varies according to regional zones of the retina (graphed in Fig. 1).

The intra-retinal nerve cells, also, are arranged in topographic zones,[3] and the nerve cells of the more peripheral retina are less dense per unit area than those of the central retina. Stimulus diffusing nerve connections including laterally conducting horizontal cells are in general more common at the periphery of the retina[3] than in the central (or foveal) region where neural mechanisms for precise stimulus transmission are better organized.

Because of the sphericity of the retinal surface,

[1] Studies with the eye camera show that when a person views a visual pattern, such as a picture, movements of the eyeball are erratic, jerky, zig-zag, and repetitive. [H. F. Brandt, Am. J. Psychol. **53**, 260–268 (1940); Am. J. Psychol. **53**, 564–574 (1940).] Even during intense fixation on a small object in the visual field, there are small erratic nystagmic movements, many per second. [F. H. Adler and M. Fliegelman, Arch. Ophthal. **12**, 475–483 (1934)].

[2] G. Østerberg, Acta Ophthal. Suppl. **6**, 102 pp. (1935).

[3] S. L. Polyak, *The Retina* (University of Chicago Press, Chicago, 1941).

it has been generally recognized[4] that only a small central region of the image is focused exactly, and that the peripheral or forward regions of the image are somewhat out of focus. This is shown by special refraction methods by which the refraction of the eye is tested over certain meridians[5] of the whole visual field. Furthermore, light passing through the eye to the more peripheral parts of the retina must traverse the pupil of the eye very obliquely so that the effective pupillary aperture subtending points on the retinal periphery is elliptical and of reduced area. The luminous flux transmitted to the retina accordingly becomes progressively less per unit area toward the periphery of the retina. Altogether, a two- to threefold difference between central and peripheral illumination intensity results. In addition, and more importantly, such light to the periphery of the retina must strike the retina at an oblique angle rather than normal to the surface. In view of the Stiles-Crawford effect[6]—light striking the retina perpendicular to its surface appears brighter than that striking at an angle—the strength of sensation at the periphery of the retina must be still further decreased. This Stiles-Crawford effect occurs with high intensity illumination but not with very low intensity illumination.

Various factors, it is seen, show a gradation from the central to the peripheral part of the image. A homogeneously illuminated regular pattern filling the whole visual field obviously forms a stimulus configuration that ranges from acuity and brightness centrally to indistinctness peripherally. Since the time of Aubert and Foerster,[7] the drop in peripheral acuity has been critically attested in living eyes. In addition, tests on living eyes, relating to sensitivity of the retina to the lowest levels of perceived brightness, for small test spots, and to color discrimination, have shown a decrease in sensitivity from the central to the peripheral regions of the retina.[8] This statement regarding sensitivity of the retina, however, is complicated by such factors of retinal physiology as: sensitivity differences associated with light and dark adaptation, variations in sensitivity to various colors, and the increase in the area of retinal sensitivity caused by high intensity illumination of test spots. As an indication, however, foveal sensitivity for the light adapted eye may be several hundred to several thousand times that of the far periphery.

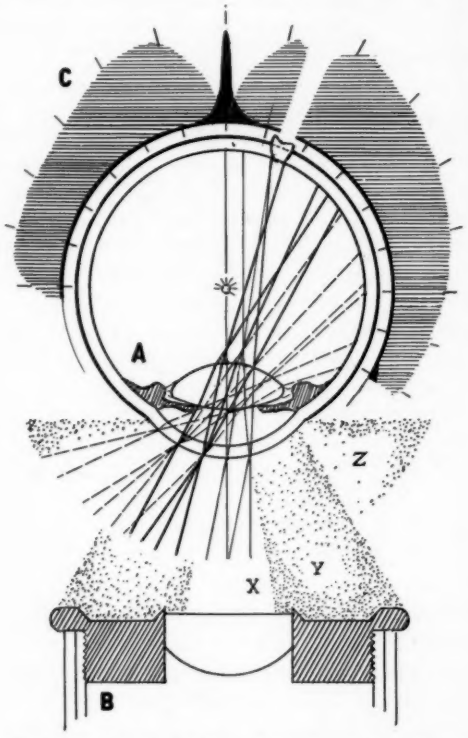

FIG. 1. The eyeball (*A*) is shown in section above the ocular of a microscope (*B*). The refraction of rays from the bright field (*X*), from the ocular rim (*Y*), and from the general surround (*Z*) is diagrammed. The relative concentration of retinal cones (black region) and rods (horizontal shading) for the horizontal meridian of the retina (Østerberg's data) is represented as a polar graph surrounding the eyeball.

Recent evidence[9] has emphasized that the eye is a cumulative organ in which considerable integration of stimuli occurs at the retinal level

[4] A. Gullstrand, *Helmholtz's Treatise on Physiological Optics* (Optical Society of America, Rochester, 1924), edited by G. P. C. Southall, Vol. 1, p. 357.

[5] H. A. Wentworth, Psychol. Monograph **40**, No. 3, 1–189 (1930). L. L. Sloan, Arch. Ophthal. **22**, 233–251 (1939). C. E. Ferree, G. Rand, and C. Hardy, Arch. Ophthal. **5**, 717–731 (1931).

[6] W. S. Stiles and B. H. Crawford, Proc. Roy. Soc. London **B112**, 428–450 (1933). W. D. Wright and J. H. Nelson, Proc. Phys. Soc. London **48**, 401–405 (1936). B. H. Crawford, Proc. Roy. Soc. London **B124**, 81–96 (1937).

[7] [H. R.] Aubert and [A.] Foerster, Archiv. f. Ophthal. (Graefe) **3** Abt. 2, 1–37 (1857).

[8] H. A. Wentworth, see reference 5.

[9] W. E. LeGros Clark, Physiol. Rev. **22**, 205–232 (1942).

as well as at the brain. Stimuli from the 6–7 million cones and the 110–125 million rods estimated[2] to be present in the human eye, are transmitted to the brain by only one million (1,010,000) optic nerve fibers.[10] Complex intraretinal associative nerve connections accumulate stimuli from adjacent zones or groups of sensitive elements forming an overlapping system—a system, however, that retains the ability of transmitting sensations of distinct imagery from the foveal region. A dynamic viewpoint toward acuity[11] derives from the modern work on the physiology of vision, and there is much to indicate that the eye is a mechanism for which all light stimuli entering the eye have significance. Foveal vision considered apart from the activity of the retina as a whole is an academic dissociation.

When the eye is centered over a microscope, the eye, of course, is erratically scanning the field. On the average, however, the image of the bright field is projected on the retina with its central 4.5° falling on a high concentration of cones and the more peripheral part falling on a region in which the cone density is low (Fig. 1). Rods, though absent from the fovea, appear in the parafovea and increase to a maximum density within the area of the retina to which the bright field image is projected. It is the area of the retina subtended by the bright field, and possibly a few degrees of subtense more, that is the zone of maximal retinal sensitivity; sensitivity decreases markedly beyond this.[5] The rim of the ocular and the general surround are projected as relatively out-of-focus images to regions of the retina characterized by a low and relatively constant cone concentration and a peripherally decreasing concentration of rods. The center of the visual field (external foveal region of the retina) is the center of visual attention while the periphery forms a background that may or may not obtrude noticeably, but which, nevertheless, has a significant relation to vision as a whole.

[10] S. R. Bruesch and L. B. Arey, J. Comp. Neurol. **77**, 631–665 (1942).

[11] S. H. Bartley, *Vision, a Study of Its Basis* (D. Van Nostrand Company, Inc., New York, 1941). W. H. Marshall and S. A. Talbot, Biological Symposia, reference 7 (Visual Mechanisms) 117–164 (1942). G. L. Walls, J. Opt. Soc. Am. **33**, 487–505 (1943).

THRESHOLD CONDITIONS FOR VISUAL DISCRIMINATION

Clear visual perception demands that an object be suitably large, in the sense of subtending an appreciably large retinal area, that it be contrasty in relation to its background, that the illumination intensity be high enough, and that a suitable time (fraction of a second) be allowed.[12] If any of these factors is below a threshold value, the object will not be seen. Very low values for any one factor may be compensated, within limits, by higher values of the other factors.

In microscopical work, the time factor is probably unimportant. Intensity, though very important, is readily adjusted by trial and error methods if illuminants of the better sort are available; intensity, thus, need not be a troublesome factor.

The numerical aperture of objectives, however, forms a limit to the effective magnification of a microscope, i.e., resolution increasing with magnification. It is often implied that a magnification more than one thousand times the numerical aperture of the objective used is empty or useless magnification. This limit, however, is arbitrary, and it should be noted that 5×, 10×, and 15× oculars do not permit magnifications that even approach the limit. Beck[13] has pointed out that high power oculars, even though they do not increase resolution, are a means of increasing the visual angle and are justified by the increased ease of seeing. In other words, visual resolution increases with further magnification even though microscope resolution has reached a maximum. With some eyes and particularly where there may be visual defects, increased magnification may be more important than with other eyes.

Of course, it is obvious that a worker will

[12] *General:* P. W. Cobb and F. K. Moss, J. Frank. Inst. **205**, 831–847 (1928). P. W. Cobb and F. K. Moss, Trans. Illum. Eng. Soc. **23**, 1104–1120 (1928). Important but more restricted studies are: *Area:* A. H. Holway and L. M. Hurvich, Am. J. Psychol. **51**, 687–694 (1938). G. Wald, J. Gen. Physiol. **21**, 269–287 (1938). W. J. Crozier and A. H. Holway, J. Gen. Physiol. **23**, 101–141 (1939). C. H. Graham and N. R. Bartlett, J. Exper. Psychol. **27**, 149–159 (1940). *Intensity:* C. H. Graham and E. H. Kemp, J. Gen. Physiol. **21**, 639–650 (1938). N. R. Bartlett, J. Exper. Psychol. **31**, 380–392 (1942). M. Keller, J. Exper. Psychol. **28**, 407–418 (1941). *Contrast:* E. Ludvigh, Arch. Ophthal. **25**, 469–474 (1941). C. Fabry, Proc. Phys. Soc. (London) **48**, 747–762 (1936).

[13] Conrad Beck, *The Microscope, Theory and Practice* (R. and J. Beck, London, 1938).

determine by a trial and error method the most useful ocular-objective combination for use with his material. Magnification, object contrast, size of field, the detail to be seen, and the general purview are compromised to suit requirements. As a practical rule, however, magnification in the sense of extent of image spread on the retina should receive due emphasis as a factor relating to ease of seeing. It would seem to be advisable to use the low power oculars (below 20× or 25×) in preference to the higher powers only when the over-all or panorama view of an object is required or when detail is secondary to general view.

Microscope images may exhibit all degrees of contrast. The objects themselves impose limitations on image contrast insofar as contrast is caused by relative opacity of objects (in relation to background brightness) or to refractive properties. Differential staining, as employed in biology, enhances contrast. Whatever contrast an object may possess, however, may be so weakened in the image by the way in which the microscope is used that good photomicrographs and clear vision are impossible. Lens aberrations of all kinds and stray light or glare (caused by lenticular reflections, dirty lenses, reflections on microscope parts, etc.) produce a misty haze that destroys contrast. Low contrast impairs visual resolution. Two points, for instance, are just visible as such under conditions of maximum contrast; with low contrast, the limiting separation compatible with two-point discrimination may be increased several times (Fabry[12]). In terms of microscopy, this is important. Under conditions of serious glare, an objective of high numerical aperture might not allow as good visual resolution as an objective of low numerical aperture but permitting good contrast.[14] Manipulation for obtaining "controlled illumination" and minimal haze, together with a rationale for these procedures, is fully treated in a separate communication.[15]

[14] Glare and decreased contrast caused by lenticular reflections, etc. in field glasses may account for results of Martin and Richards that appear contrary to the weight of evidence to be presented in the next section. A field glass was adjusted for a wide view angle and for a narrow angle and these were compared at high and low intensities for the perception of small dark objects on a bright field. Higher acuity was found with low intensity and wide field or with high intensity and narrow field; high intensity and wide field must have been inefficient because of glare. [L. C. Martin and T. C. Richards, Trans. Opt. Soc. London **30**, 22–33 (1928–29).]

[15] W. T. Dempster, J. Opt. Soc. Am. **34**, 695 (1944).

In practical microscopy, the most effective vision may be obtained only through the best possible combination of the pertinent factors. Benefits may be obtained through: 1. use of the higher power oculars as a routine in conjunction with the various objectives; 2. adjustment of illumination intensity till it is optimal; 3. avoidance of veiling glare; 4. attention to the conditions for obtaining maximum contrast;[15] and 5. a knowledge of the factors (below) that increase visual acuity.

FACTORS CONCERNED IN ACUITY

It is generally recognized that there is a critical boundary between things seen or differentiated and things beyond the powers of visual resolution. The measure of this transition in relation to size, brightness, etc., is called acuity. Types of acuity as defined for purposes of scientific exploration are ably reviewed by Walls.[11] A group of experiments relating to the effect of the visual periphery on the acuity of central vision provides a picture of visual efficiency that has practical implications for the microscopist. Investigators, using controlled experimental conditions and the techniques of psychophysical testing, have examined the acuity of subjects for the ability to discriminate brightness differences,[16] for the ability to distinguish details of gratings and test figures (C, ||, etc.)[17] and for the recognition of flicker or light fusion in spot sources having variable flicker rates.[18]

When surroundings are brighter than the small test objects, acuity is decreased. Similarly, surrounds noticeably darker decrease acuity. When the surround and test object are approximately equally illuminated, acuity is best. Larger sur-

[16] P. W. Cobb, J. Exper. Psychol. **1**, 540–566 (1916). J. Steinhardt, J. Gen. Physiol. **20**, 185–209 (1936). Holway and Hurvich, reference 12. M. Luckiesh and F. K. Moss, Trans. Illum. Eng. Soc. **34**, 571–597 (1939). S. H. Bartley and G. A. Fry, J. Opt. Soc. Am. **24**, 342–347 (1934).

[17] P. W. Cobb and L. R. Geissler, Psychol. Rev. **20**, 425–447 (1913). R. J. Lythgoe, "The measurement of visual acuity," Privy Council Medical Research Council Spec. Rep. Ser. No. 173, 85 pp. (1932). S. Shlaer, J. Gen. Physiol. **21**, 165–188 (1937). M. B. Fisher, J. Exper. Psychol. **23**, 215–238 (1938).

[18] R. J. Lythgoe and K. Tansley, "The adaptation of the eye; its relation to the critical frequency of flicker," Privy Council Medical Research Council Spec. Res. Ser. No. 134, 72 pp. (1929). S. Hecht and E. L. Smith, J. Gen. Physiol. **19**, 979–989 (1936). Perception of flicker is probably not pertinent to microscope vision, but it is of interest to note that this faculty is influenced by peripheral illumination much the way that acuity discrimination is.

rounds, if suitably illuminated, are better than small. Essentially the same effects on brightness difference discrimination are noted[19] when the eye is preadapted to fields that are brighter, equally bright, or darker than the test object as when the field and object are exposed simultaneously.

Where color detection is concerned, pre-exposure and surrounding fields of an intensity of grey equal to that of test colors are essential if retinal sensitivity to color is to be increased.[20] Noticeably lighter or darker backgrounds or pre-exposure causes contraction of the size of the color sensitive fields of the eye.

Spot glare sources peripheral to the test object often cause a decrease in acuity and their pernicious effect becomes greater as the glare source comes closer to the visual axis and becomes less, though remaining perceptible, toward the retinal periphery.[21] The more intense or the larger the glare source, the greater becomes the detriment to vision. The obfuscating effects of glare are in part probably caused by the scattering of light within the eyeball[22]—which will decrease the contrast of the retinal image at the foveal region—but there is clear evidence[23] that foveal sensitivity is actually decreased by an adaptive influence arising at the site of the glare stimulus and spreading through retinal neurones over the whole retina. Such an inhibiting mechanism might account also for the observation, noted above, that large dark surrounds decrease central acuity and the finding[24] that dark rings projected on to the retinal surface likewise diminish acuity. Under special conditions—contrasty test objects and dark surrounds—spot glare sources may actually enhance central acuity.[25] For instance, Sewall[26] long ago indicated that in certain types of critical astronomical work with the telescope, better vision was obtained if a lamp was nearby than if the workroom was dark.

Broadly interpreted, in relation to a retinal field having a gradient of increasing diffuseness from the center to the periphery, the observations referred to show that central acuity is in large part dependent on an optimal illumination of the whole retinal surface. Marked discontinuities of illumination—glare, dark areas, constriction of the field—decrease central acuity. The detriment to acuity becomes greater as the discontinuities encroach on the visual axis, as they become larger or as the intensity of the peripheral discontinuity becomes greater.

VISUAL COMFORT

In addition to the type of work discussed, the general conditions of illumination have been studied by a purely objective test[27]—the rate of blinking at the onset and at the termination of an hour's reading. Discomfort may be measured by a significantly increased blink rate. Glare sources[28] and surrounding illumination that is either brighter or darker than the book page increases the rate of blinking over that noted when the surround is homogeneous and of an intensity matching that of the page.

It is of interest to note that the peripheral conditions for visual comfort and those for high visual acuity as well are essentially the same. Basic work on the conditions affecting visual capacities becomes convincing since it is both consistent and reasonably ample. It provides a background for the lighting engineer in his adjustment of environments for high visual comfort and efficiency. In terms of retinal illumination, comfort and foveal efficiency are high when the general illumination is relatively homogeneous and of a fair level of intensity. Objects to be discriminated with ease should be reasonably large and should have high contrast in relation to the background. Discontinuities in peripheral homogeneity (i.e., glare spots, constriction of the visual field, or large dark areas projected to the retina) are deleterious.

PRACTICAL APPLICATIONS

With this in mind, it becomes obvious that the microscope does not provide the superior

[19] K. J. W. Craik, J. Physiol. **92**, 406–421 (1938).
[20] C. E. Ferree and G. Rand, Psychol. Rev. **27**, 377–398 (1920). C. E. Ferree and G. Rand, Am. J. Ophthal. **7**, 843–850 (1924).
[21] Lythgoe, see reference 17.
[22] S. H. Bartley, J. Comp. Psychol. **19**, 149–154 (1935).
[23] J. F. Schouten and L. S. Ornstein, J. Opt. Soc. Am. **29**, 168–182 (1939).
[24] A. T. Chuprakov, Vestnik. Oftal. **17**, 680–685 (1940). From Biol. Abs. 16 (1942), Abs. 19979.
[25] Lythgoe, see reference 17.
[26] H. Sewall, J. Physiol. **5**, 132–139 (1884).
[27] M. Luckiesh and F. K. Moss, J. Exper. Psychol. **20**, 589–596 (1937). M. Luckiesh and F. K. Moss, Am. J. Ophthal. **22**, 616–621 (1939). M. Luckiesh and F. K. Moss, Trans. Illum. Eng. Soc. **35**, 19–32 (1940).
[28] M. Luckiesh and F. K. Moss, J. Opt. Soc. Am. **32**, 6–7 (1942).

conditions for effective visual performance that might be expected in a precision instrument. The ocular diaphragm, the dark interior of the ocular, and the darkened rim form a contrasting annulus around the bright field and constitute an undesirable dark surround. Beyond the ocular (but within the visual field), there may be glare sources at the work table and illumination-intensities that would preclude visual efficiency.

Present-day oculars, apart from superior lenses and wider fields, are little different from those of the last century. No doubt better oculars for visual work could be designed if attention were directed to eliminating the dark annulus surrounding the bright field image of the microscope. This is partially achieved in high eye-point oculars, but it may be that improvement would result if the eye lenses of the ocular were supported in a transparent or translucent ocular rim made of plastic and if the peripheral rays of the beam passing through the draw tube were used to illuminate the translucent ocular rim. Possibly, tubular ocular inserts of transparent plastic or glass would be required to refract or reflect light to the ocular rim.

A rough approximation is effected by: 1. enlarging the ocular diaphragm in a lathe till only a narrow shelf capable of supporting ocular grids remains, 2. substituting as a field boundary a transparent plastic disk bored with a central aperture the size of the original diaphragm aperture, and 3. painting the ocular rim a dull white. Such oculars provide a fair approach to a light surround for the bright field image. Obviously, oculars of this sort would afford no advantage in photomicrography or in dark field work.

Attention to the walls near the work space and to the work table top is also important. These may be coated or painted light and illuminated from above with light of suitable intensity (Fig. 2). Potential glare sources—unenclosed illuminating sources, facing windows, and reflecting metal or glass surfaces—should be avoided. The metal and black surfaces of the microscope itself are unimportant since the ocular rim precludes them from the visual field. The conditions sought are succinctly stated by Moon:[29] "The ideal lighting system will have a uniformly luminous work area, a surround which is also essentially uniform in luminosity, and an entire absence of spots of high intensity in the surround." Luckiesh and Moss[30] express the same idea.

Though the system of lighting shown in Fig. 2 is presented in the light of its value to microscopy, it should be apparent that the same lighting will be of equal value for many types of laboratory work. Obviously, the increased brightness over that of the ordinary laboratory illumination will demand several minutes adaptation before full comfort and efficiency are obtained.

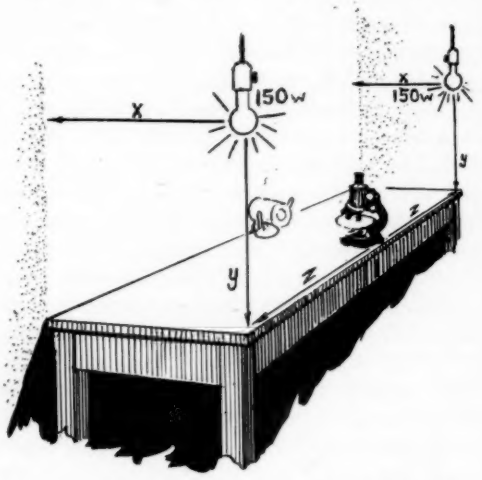

FIG. 2. A diffuse reflected illumination of about 10 eq. ft.-c is obtained with this arrangement: 2 150-w lamps without reflectors or 2 100-w lamps with reflectors; dimensions x and $y = 30''$, $z = 36''$; table top and wall a dull white. Shields (not shown) should be placed between the worker's head and the lamps to prevent direct glare.

Two eyes are better than one[31] and the binocular microscope should be preferred. Insofar, however, as the acuity of monocular vision may be improved somewhat by suitable intensity illumination for the unoccupied eye, owing both

[29] P. Moon, *The Scientific Basis of Illuminating Engineering* (McGraw-Hill Book Company, Inc., New York, 1936).

[30] M. Luckiesh and F. K. Moss, *The Science of Seeing* (D. Van Nostrand Company, Inc., New York, 1937). It will be apparent that this is a general statement relating to so-called "normal" eyes. Actually, the best selected "normal" eyes may in certain cases be defective according to special tests [I. Mann and D. Archibald, Brit. Med. J., pp. 387–390 (1944)] and optimal lighting for a specific person may depart slightly from an average optimum.

[31] C. E. Ferree, G. Rand, and E. F. Lewis, Trans. Illum. Eng. Soc. **29**, 296–313 (1934). Crozier and Holway, see reference 12.

to the consensual pupillary reflex[32] and adaptational effects on the occupied eye, a light colored and suitably illuminated work space (Fig. 2), or an opalescent screen for the unoccupied eye,[33] is indicated.

There is an optimal minimal intensity of illumination for effective vision (about 10 ft.-c[34]). This may readily be exceeded if desired since individual requirements differ. In general, acuity or resolving power increases with the intensity of microscope illumination.[35] This is a general property in the viewing of dark objects on bright backgrounds,[36] but it does not hold consistently for bright objects on dark backgrounds (i.e., the visual picture in dark field microscopy). Lythgoe[37] indicates that the continued rise in visual acuity for test objects above 38.9 foot-candles is almost certainly caused by an improvement in retinal sensitivity and not merely by reduced aberrations of the eye[38] caused by a smaller pupil, since pupil diameter is not further decreased with higher illumination. In microscopy, the pupil size of the subtending eye may be ignored because the exit pupils of oculars are so small. Increased acuity must be caused by increased sensitivity.

In general, older eyes (40+ years) require greater illumination than those of younger persons (20–30 years) for equivalent acuity values.[39] Some adjustment of intensity is called for when objectives or oculars are changed in a microscope, and the nature of objects may temporarily call for more or less light. Obviously, some means of controlling the intensity of microscope illumination (neutral wedges, filters, rheostat, etc.) is desirable so that one's intensity preference may be determined by trial and error. The condenser of a microscope should not be misused as a means of adjusting field intensity. In better microscopic practice, the condenser is used as an adjunct in procuring "controlled illumination" and special methods (filters, neutral wedges, etc.) must be used for regulating intensity. The importance of intensity control cannot be overemphasized.

There are clear indications that light of the blue and red ends of the spectrum is visually inefficient[40] and that green or yellow-green light is as good or better than white light of the same intensity. For at least sixty years,[41] some microscopists have recommended green light in preference to white for increased visual resolution. Green (yellow-green, 0.550–0.560 micra[42]) is the color of maximum spectral sensitivity. That is, for a given energy output a yellow-green light appears brighter than one of longer or shorter wave-length. The latter may require several times the intensity to appear as bright as the yellow-green. Red or blue and violet stains, so common in tissue microscopy, show color cancellation when yellow-green light is used in microscopy, and details are accentuated because of the contrast elicited. Minute details apparent through refractive differences likewise would appear more prominently in light approaching a monochromatic quality than in white light. With this light, the possibility of chromatic aberration in the microscope and eye is largely eliminated so that the achromatic microscope becomes nearly as effective as the apochromatic instrument. With strictly monochromatic light, in contrast to that generally provided by filters, it would be as efficient. Belling[33] indicates, in addition, that a series of green filters graded in transmission values may be used in combinations for the regulation of illumination intensity.[43]

[32] There is evidence that illumination of one eye in addition improves the sensitivity of the other. [G. W. Hartmann, J. Exper. Psychol. **16**, 383–392 (1933); S. V. Kravkov, J. Exper. Psychol. **17**, 805–812 (1934).] The effect appears to be more marked when the percipient eye views dark objects on a light field—an approximation to conditions in bright-field microscopy.

[33] John Belling, *The Use of the Microscope* (McGraw-Hill Book Company, Inc., New York, 1930).

[34] L. G. H. Hardy, Trans. Illum. Eng. Soc. **29**, 364–384 (1934). L. T. Troland, Trans. Illum. Eng. Soc. **26**, 107–196 (1931). R. J. Lythgoe, "Illumination and visual capacities," Privy Council Medical Research Council Spec. Rep. Ser. No. 104, 80 pp. (1926).

[35] K. B. Merling-Eisenberg, Nature **139**, 416–417 (1937). B. P. Ramsey, E. L. Cleveland, and W. A. Bowen, Jr., J. Opt. Soc. Am. **32**, 288–292 (1942).

[36] W. W. Wilcox, J. Gen. Psychol. **15**, 405–434 (1936).

[37] Lythgoe, see reference 17.

[38] P. W. Cobb, Am. J. Physiol. **36**, 335–346 (1914–15).

[39] Ferree, Rand, and Lewis, reference 31.

[40] M. Luckiesh, Elec. World **58**, 1252–1254 (1911). H. E. Roaf, Proc. Roy. Soc. London **B106**, 276–292 (1930). S. Shlaer, E. L. Smith, and A. M. Chase, J. Gen. Physiol. **25**, 553–569 (1942). C. E. Ferree and G. Rand, J. Aviation Med. **13**, 193–200 (1942).

[41] T. W. Engelmann, Arch. f. d. ges. Physiol. **23**, 505–535 (1880).

[42] H. V. Walters and W. D. Wright, Proc. Roy. Soc. London **B131**, 340–361 (1942).

[43] The author wishes to express his obligation to Dr. G. L. Walls, of the Bausch and Lomb Optical Company, and Dr. O. W. Richards, of the Spencer Lens Company, whose comments and criticism of the manuscript have been most valuable.

The Physical and Photochemical Basis of Visual Resolving Power

Part II. Visual Acuity and the Photochemistry of the Retina*

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INTRODUCTION

IN Part I of this paper, equations were derived giving the distribution of illumination in the retinal or physical images of different kinds of test objects. From these distribution equations it was possible to derive visual acuity equations for certain objects at high brightness levels which expressed visual acuity as a function of the contrast between the objects and their backgrounds. These equations were in good agreement with visual acuity data. The data for single lines showed that visual acuity was closely related to brightness discrimination,** and calculations based on the precise data of Shlaer,[1] and Shlaer, Smith, and Chase,[2] indicated that this might also be true for the grating test object. However, the data for individual disks seemed to show that visual acuity at high brightness levels was determined by a constant flux differential on the retina. If threshold visual acuities are determined by small but constant flux differentials rather than by constant illumination differentials, it would appear that integrative interaction, or summation, exists among the receptors because the threshold contrast decreases as the area of the image increases. However, to account for the high resolving power of the fovea, it is necessary to assume that each foveal cone has a highly insulated pathway to a fixed location in the visual cortex of the brain. This assumption is not compatible with the concept of summation and seems to contradict the observed effects which borders and backgrounds exert on visual acuity. The distribution of light in retinal images shows that the borders of test objects should appear blurred, yet for normal pupil apertures they appear sharp and well defined.

The visual image received at the visual cortex of the brain is usually referred to as the mental, subjective, or cortical image. Some investigators believe that certain properties of the cortical image, such as the "sharp" contours of test objects, cannot be explained at the retinal level and to account for them have postulated neural mechanisms operating in the post-retinal system. Perhaps these mechanisms are helpful in explaining some phases of the visual process, but it is doubtful whether they are necessary for explaining the data of visual acuity and contour recognition which apparently can be explained in a simpler manner at the receptor level in terms of physical and quantum optics and the photochemistry of the retina. In this paper it will be shown that if the concept of retinal illumination discrimination is replaced by the concept of the discrimination of differences in the rates at which the receptors convert luminous energy into energy of some other form (which can produce a visual sensation), then most of the phenomena of visual acuity and contour recognition can be explained in terms of the diffraction theory of image blurring and receptor chemistry. However, the treatment of some of the data of visual acuity, such as area-threshold effects, requires the use of the quantum theory and this phase of vision will be discussed in a later paper.

THE RESOLVING POWER OF THE RETINA

The photochemical aspects of visual acuity can be more clearly understood by first considering the function of the retinal mosaic in resolving power acuity and perception acuity. In some respects the retina resembles a photographic film of very fine grain and, as in the case of the film, one would expect the finite nature of the "grain" of the retina to have some effect on its

* Paper presented at the Winter Meeting of the Optical Society of America, held in New York City, March 2–4, 1944.

** Some investigators use the terms "brightness discrimination" and "intensity discrimination" interchangeably. Probably a better term would be "retinal illumination discrimination," because the distribution of illumination in the retinal images of very small test objects may differ considerably from the distribution of brightness in the test objects.

[1] Simon Shlaer, J. Gen. Physiol. **21**, 165–188 (1937).

[2] Simon Shlaer, E. L. Smith, and A. M. Chase, J. Gen. Physiol. **25**, 553–569 (1942).

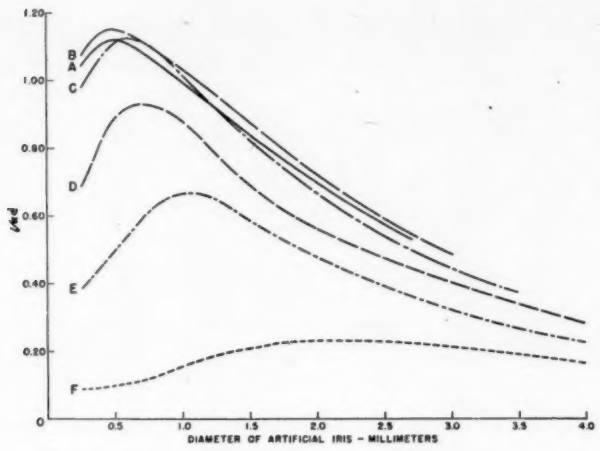

FIG. 1. Curves showing $1/\alpha d$ as a function of the artificial iris aperture at different brightness levels for black parallel bars on a white ground. The brightness H_b of the white background for the different curves was: A—7000 millilamberts, B—700 millilamberts, C—70 millilamberts, D—7 millilamberts, E—0.7 millilambert, F—0.07 millilambert.

image forming qualities. Histological investigations show that the foveal cones have a diameter of about 2.5 microns, or an angular diameter of about 0.5 minute. If one assumes that the cones are solidly packed in this area, this figure represents approximately the distance between cone centers. Each foveal cone has its own fiber in the optic nerve, thus giving a one-to-one correspondence between this area of the retina and the corresponding part of the visual cortex. However, the angle of critical vision is very small (less than 1°) and the structure of the outer retina is much less favorable for high visual acuity. At increasing distances from the fovea the density of the cone population decreases and the rods, the receptors which take over the function of vision in very faint light, increase in number. Many rods may be associated with a single nerve fiber which probably decreases the absolute brightness threshold for low brightness levels but accomplishes this at the expense of distinct vision.

The effect of the finite structure of the foveal cone mosaic on visual resolving power can be determined by varying the size of the basic diffraction pattern. In a telescopic system, the images of two distant self-luminous point sources of light are considered resolved when the central maximum of the diffraction pattern of one image falls on the first minimum of the pattern of the other image. When these conditions are fulfilled, the angular separation α (in radian measure) of the sources is given by the familiar equation

$$\alpha = 1.22\lambda/d, \qquad (1)$$

where λ is the wave-length of light, and d the diameter of the telescope objective. This is a physical definition of resolution known as the Rayleigh criterion. It is an arbitrary but useful definition chosen because of its physical and geometric simplicity, but it does not necessarily follow that, if this equation is applied to the human eye, point sources with the angular separation indicated by Eq. (1) can be subjectively resolved. Cobb's measurements[3] indicate that α actually increases when d is greater than 4 mm, or when spherical and chromatic aberration become appreciable. However, for apertures below 2.5 mm, diffraction accounts for most of the blurring of retinal images provided that the retinal contrast u is small. In Part I of this paper it was shown that the retinal contrast u in the image of a given test object is a function of the object-background contrast v and some arbitrary angular dimension of the test object. This angular dimension may be expressed in terms of a reduced angular unit z_a, or in terms

[3] P. W. Cobb, Am. J. Physiol. **36**, 335–346 (1915).

TABLE I. Visual acuity data for different values of the background brightness H_b and different artificial iris apertures d. All data are for the Luckiesh-Moss test object.

$H_b \rightarrow$	7000		700		70		7		0.7		0.07	
d	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$
0.277	3.40	1.06	3.34	1.08	3.67	0.980	5.50	0.658	9.18	0.394	36.8	0.098
0.463	2.00	1.08	1.84	1.17	1.90	1.14	2.34	.926	3.94	.550	22.0	.098
0.527	1.64	1.16	1.62	1.17	1.77	1.07	2.00	.952	4.23	.449	22.0	.086
0.612	1.49	1.09	1.49	1.10	1.41	1.16	1.83	.893	3.06	.534	15.8	.103
0.686	1.38	1.06	1.33	1.10	1.27	1.15	1.53	.952	2.56	.568	15.8	.093
0.756	1.28	1.03	1.22	1.08	1.25	1.06	1.47	.900	2.12	.625	11.0	.120
0.813	1.11	1.11	1.13	1.08	1.10	1.12	1.38	.893	1.78	.690	9.18	.134
0.929	1.03	1.04	1.00	1.07	1.02	1.06	1.17	.917	1.90	.565	6.88	.156
1.17	0.911	0.943	0.860	0.995	0.901	0.952	0.991	.862	1.29	.662	4.99	.172
1.41	0.781	0.909	0.788	0.901	0.829	0.862	0.984	.725	1.07	.666	3.14	.226
1.63	0.754	0.820	0.709	0.870	0.764	0.807	0.939	.654	1.15	.534	3.23	.190
2.00	0.732	0.685	0.688	0.730	0.753	0.666	0.918	.546	1.03	.485	2.00	.251
2.41	0.678	0.613	0.685	0.606	0.743	0.558	0.846	.490	1.01	.410	1.80	.230
3.23				0.448	0.743	0.418	0.798	.387	1.08	.287	1.62	.191
3.97							0.901	.279	1.10	.229	1.45	.174

of the angular unit α (either in radians or minutes). The threshold value of u is written as u_t, and when v is unity the equation for u_t in terms of z_a is

$$u_t = f(z_a). \tag{2}$$

For some test objects the function $f(z_a)$ can be most conveniently expressed as a power series. If the limit of visual resolving power is determined by retinal illumination discrimination, then for any given retinal illumination u_t should be constant, and from Eq. (2) it follows that z_a should also be constant. However, z_a is given by the equation

$$z_a = \pi d\alpha/\lambda,$$

which, if z_a is constant, may be written as

$$1/\alpha d = k/\lambda, \tag{3}$$

where $1/\alpha d$ is the visual acuity per unit of iris aperture and k is a constant. Equation (3) is a generalized form of Eq. (1) and since it applies to test objects other than two point sources of light, it can be used to test the retinal illumination discrimination theory for any number of objects.

Figure 1 shows the results of a series of visual acuity tests with a test object identical in form to that used by Luckiesh and Moss.[4] This object

[4] Matthew Luckiesh and F. K. Moss, *The Science of Seeing* (D. Van Nostrand Company, Inc., New York, 1937).

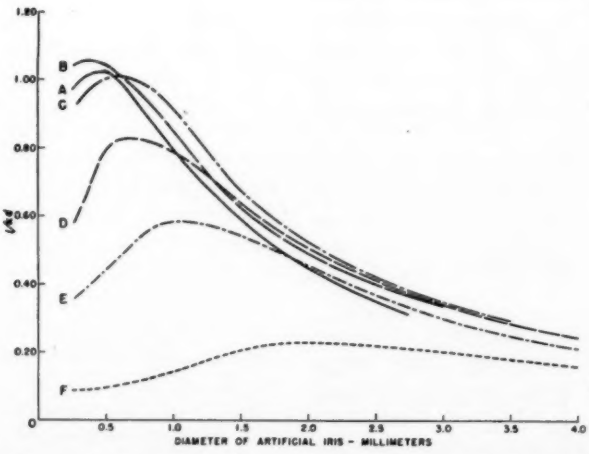

FIG. 2. Curves showing $1/\alpha d$ as a function of the artificial iris aperture at different brightness levels for white parallel bars on a black ground. The brightness H of the white bars for the different curves was the same as for the backgrounds in Fig. 1.

TABLE II. Visual acuity data for different values of the test object brightness H and different artificial iris apertures d. Data are for a test object complementary to that used for the data in Table I.

$H \rightarrow$	7000		700		70		7		0.7		0.07	
d	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$	α	$1/\alpha d$
0.277	3.78	0.952	3.40	1.06	3.89	0.926	6.50	0.556	9.74	0.370	38.9	0.093
0.463	1.95	1.11	2.02	1.07	2.13	1.01	2.49	.869	4.68	.461	19.5	.111
0.527	1.92	0.990	2.02	0.944	1.98	0.961	2.25	.840	4.51	.420	23.4	.081
0.612	1.70	0.961	1.67	0.980	1.60	1.02	2.09	.781	3.89	.420	16.7	.098
0.686	1.58	0.926	1.45	1.01	1.39	1.05	1.72	.848	2.72	.537	14.6	.100
0.756	1.39	0.952	1.43	0.926	1.34	0.990	1.67	.794	2.34	.565	10.6	.125
0.813	1.34	0.918	1.31	0.944	1.24	0.990	1.56	.788	1.98	.621	10.6	.116
0.929	1.22	0.886	1.22	0.885	1.17	0.917	1.31	.820	2.25	.478	6.88	.156
1.17	1.19	0.719	1.09	0.788	1.01	0.847	1.14	.752	1.44	.595	5.85	.146
1.41	1.07	0.663	1.13	0.629	1.02	0.694	1.08	.658	1.23	.578	3.16	.225
1.63	1.11	0.556	1.04	0.592	0.936	0.658	1.05	.585	1.22	.505	3.16	.195
2.00	1.19	0.420	1.04	0.481	0.980	0.510	0.991	.505	1.09	.458	2.12	.236
2.41	1.07	0.387	0.96	0.431	0.936	0.442	0.973	.426	1.08	.383	2.02	.204
3.23				0.317	0.949	0.326	0.915	.338	1.15	.269	1.65	.187
3.97							1.02	.247	1.17	.216	1.46	.172

consists of two black parallel bars on a white ground separated by a space equal to their width which is one-third their length. Visual acuity is the reciprocal of the angular separation α (in minutes) between the inner edges of the bars when they are just resolved. Values of $1/\alpha d$ are plotted as a function of the artificial iris aperture d which varied from 0.277 mm to 3.96 mm. The $1/\alpha d$ curves were determined for six different values of the background brightness H_b ranging from 0.07 millilambert to 7000 millilamberts. To avoid crowding, individual measurements are not shown but are presented in Table I instead. The curves show several important properties; the quantity $1/\alpha d$ (which is a measure of the resolving power efficiency of the retina) is a maximum when d is about 0.5 mm, or when the diameter of the diffraction disks on the retina is about 20 times that of the foveal cones. The sharp decrease in $1/\alpha d$ at small apertures might be attributed to low retinal illuminations, but if the data are plotted for constant retinal illuminations rather than for constant object brightnesses, the form of the curves remains substantially the same although all maxima come at about the same value of d.

Figure 2 and Table II show similar data for a test object (white bars on a black background) complementary to the Luckiesh-Moss object. The brightness H of the white bars was the same as that of the background in Fig. 1. The curves for both test objects are similar at low brightness levels but differ considerably at high brightness levels. From either Fig. 1 or Fig. 2 it is obvious that even for small iris apertures Eq. (3) does not apply to visual resolution. Since the quantity $1/\alpha d$ is not constant at the limit of resolution, then z_a and u_t cannot be constant, and the illumination discrimination theory is not valid for the parallel bar test object.

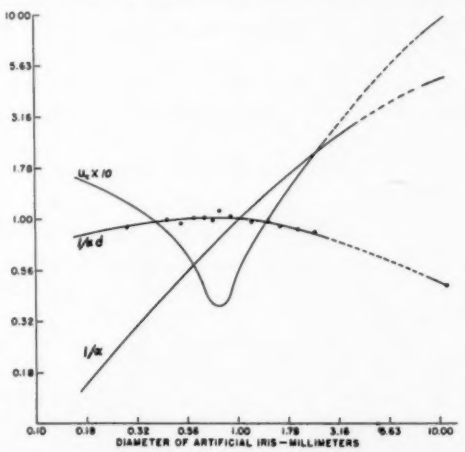

FIG. 3. The curves $1/\alpha d$, u_t, and $1/\alpha$ for the grating test object are shown on logarithmic scales as a function of the artificial iris aperture d. Values of the curves for apertures greater than 2.5 mm were obtained by observing a bright line source through a double-slit diaphragm. To bring all curves to approximately the same position, the u_t curve is multiplied by a factor of 10.

The $1/\alpha d$ curves were determined over an extended brightness range for only the parallel bar test object, but tests made for the readability of typewritten print gave an almost identical curve for a brightness of 70 millilamberts. Figure 3 shows the $1/\alpha d$ curve for a grating test

object in which the width of the bright lines is small in comparison with the width of the interspaces. The mean brightness over the whole grating (including interspaces) was 3000 millilamberts. The angle α is the angular distance between the center of a white line and the center of an adjacent black line. The flatness of the $1/\alpha d$ curve is somewhat misleading and for the grating test object, a curve showing the retinal threshold contrast u_t plotted as a function of d (Fig. 3) is a better measure of the efficiency of the retina than the $1/\alpha d$ curve. In general this is true for most test objects but for many objects u_t is difficult to compute, because it requires the evaluation of $f(z_a)$ in Eq. (2). The visual acuity $1/\alpha$ is also shown as a function of d in Fig. 3.

Because of spherical aberration, it is impossible to obtain accurate curves of u_t for apertures greater than 2.5 or 3.0 mm by means of a circular pupil. However, this can be achieved virtually for the grating test object by means of the double slit. If a distant line source of monochromatic light is viewed through a double-slit diaphragm held so that the slits are parallel to the line source, the source will appear as a luminous rectangular shaped area traversed by a number of very sharp bright and dark lines. The distribution of the illumination I in these lines is given by the familiar equation of the double slit

$$I = I_0(\sin z_1/z_1)^2 \cos^2 z_2, \qquad (4)$$

where I_0 is the illumination at the center of the central bright line and z_1 and z_2 are defined by the equations

$$z_1 = \pi a\phi/\lambda, \quad z_2 = \pi b\phi/\lambda, \qquad (5)$$

where a is the width of the slits, b the distance between the centers of the slits, and ϕ the angular distance (measured from the posterior node of the eye) of a given point in the retinal image from the center of the central bright line. When ϕ is large it must be replaced by $\sin \phi$, but in this experiment it is always very small (less than 10 minutes). The distribution of light in the double-slit diffraction pattern on the retina is almost identical with the intensity distribution in the image of a grating viewed through a circular aperture provided that the spacing of the lines in both images is equal and if the ratio between the diameter d of the aperture and the distance b between the slits is such that

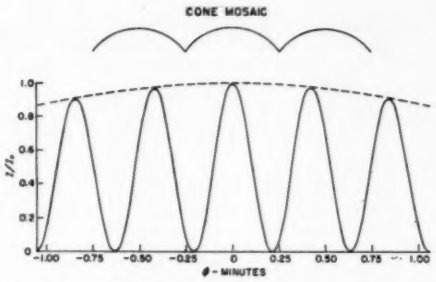

FIG. 4. The distribution of illumination in the retinal image of a bright line source of light viewed through a double-slit diaphragm for which $a = 0.3$ mm and $b = 4.5$ mm. The dotted line is an envelope representing the single-slit pattern.

$$d = 2.36b. \qquad (6)$$

A double slit with $b = 4.5$ mm thus has the same resolving power, or rather is capable of producing a diffraction pattern of as fine a structure, as a circular aperture of about 10.6 mm. Equation (4) is plotted in Fig. 4 for slits 0.3 mm wide* and a 4.5-mm separation between slit centers. Such a pattern can just be resolved by the retina yet the cone mosaic scale shows that the distance between adjacent maxima and minima ($\alpha = 0.2$ minute) is considerably less than the distance between centers of adjacent cones. Maxima cone at $z_2 = k_2\pi$ where k_2 is any integer, so the angular distance α between the center of a bright line and the center of an adjacent dark line is

$$\alpha = \lambda/2b.$$

Certain bright lines or orders will be missing if they happen to come where $z_1 = k_1\pi$, k_1 being any integer except 0, because of the sin factor, but this is of little consequence since the first two or three lines on each side of the bright central image are sufficient for observation.

When a bright line on a dark background is observed through the double slit, the contrast u in the retinal image is unity. If the line is observed against a background the brightness of which can be varied, the contrast can be given any value between zero and unity. By determining the background brightnesses which cause

* For the actual observation of the bright and dark lines, the width of the slits should not exceed 0.1 millimeter.

the lines in the diffraction pattern to disappear for various values of α, the threshold contrast u_t can be determined for all values of b between 0 and 4.5 mm. However, if b is less than 1.06 mm, which is equivalent to $d=2.5$ mm, u_t can be determined more easily by observing the grating test object through circular apertures of various diameters. The curves in Fig. 3 are all plotted as a function of d rather than b. With the double slit the threshold value of α was determined only for $u_t=1$, so data are missing in the dotted parts of the curves. The trend of the u_t and $1/\alpha d$ curves shows that grating resolution cannot be satisfactorily accounted for by a retinal illumination discrimination theory.

Helmholtz noticed that near the limit of resolution the lines in a grating took on a wavy appearance. The writer has never noticed this for a circular aperture, but it is clearly shown by the double-slit experiment. If observations are started with $\alpha=0.75$ minute, the lines in the diffraction pattern are very sharp and distinct; if b is increased so that $\alpha=0.45$ minute, the lines appear as sharp as before, but are more difficult to see. When $\alpha=0.35$ minute the lines are still visible but begin to undergo a surprising change in appearance. They no longer appear straight and parallel but have a definite wavy appearance and are subject to a rapid shimmering or fluttering movement. The wavy appearance apparently is due to the retinal structure and the observer is actually "seeing" the cone mosaic. When $\alpha=0.2$ minute, the lines are barely visible and become broken up into a series of wriggling curved line segments which are rapidly changing their shape and position. The apparent movement of these segments probably is due to the small but rapid eye movements described by Marshall and Talbot.[5] Figure 5 shows roughly the appearance of the lines for three different values of α. Shlaer[1] assumed that the angular distance between a line of maximum intensity and a line of minimum intensity in the image of a grating could not be less than the distance between centers of adjacent rows of foveal cones, and that the maximum visual acuity obtainable with the grating could not exceed 2.0 or 2.1 minutes^{-1} if the angular distance between adjacent cone rows is approximately 0.5 minute. Figure 5 shows that the limit to the resolving power of the retina is ultimately determined by the cone mosaic but that the limiting visual acuity is about 5.0 minutes^{-1} provided that one is willing to accept the curved and broken lines as an image. If it is required that the grating lines appear straight and parallel, then the limiting visual acuity is near Shlaer's value.

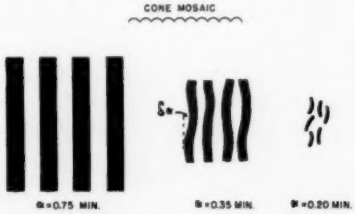

FIG. 5. These drawings show the appearance of the lines in the image of a line source viewed through a double-slit diaphragm for $\alpha=0.75$, 0.35, and 0.20 minute. When $\alpha=0.20$, the lines become broken up into discontinuous curved segments.

From a practical standpoint the curved and broken line segments of Fig. 5 can hardly be considered as equivalent to the image of a grating test object, and the resolving power of the retina probably should be computed for an image in which the angular spacing of the lines is such that the irregularities are just noticeable. This occurs when the angular distance between a maximum and an adjacent minimum is about 0.35 minute. If the focal length of the eye is 16 mm, it follows that the resolving power of the retina (in the central fovea) is about 300 lines per mm, or about twice as great as that for a fine grain photographic film.

The curves in Fig. 3 seem to describe in a fairly complete manner the role which the retina plays in the process of visual acuity, at least for the grating test object. If the aperture of the eye could be made much larger than it actually is, and if the optical system were free from spherical and chromatic aberration, the visual acuity $1/\alpha$ would increase with d until an aperture of 10.6 mm was reached, and beyond this the increase would be negligible. The value of this limiting aperture can be estimated in another way. Since the central diffraction disk in the image of a point source of light contains about 84 percent of the total flux in the image, then

[5] *Visual Mechanisms*, edited by Heinrich Kluver (The Jaques Cattell Press, Lancaster, Pennsylvania, 1942), Chapter VII.

making the diameter of this disk smaller than that of an individual cone (that is, smaller than 0.5 minute) could contribute little to the effective contrast in the retinal image of any given test object. For the diffraction disk to have a diameter of 0.5 minute, the diameter of the iris aperture would have to be 9.4 mm, which agrees fairly well with the value of 10.6 computed from the results of the double-slit experiment. Actually the iris cannot exceed 8 mm and aberrations at this aperture prevent good definition. Moreover, the work of Stiles and Crawford[6] indicates that the outer zones of the pupil are less efficient for illuminating the receptors than are the central zones.

The visual acuity curve $1/\alpha$ gives no information as to the efficiency of the retina which can be found from either the u_t or $1/\alpha d$ curves. The curve of the retinal threshold contrast u_t has its minimum value when $1/\alpha d$ is a maximum. The latter curves probably would be useful in determining the magnification for which a microscope or telescope will reveal the greatest detail. It can be shown that either of these should reveal the greatest detail when the magnification is such that the diameter of the pencil of light emerging from the eyepiece is equal to the value of d which corresponds to the maximum of the $1/\alpha d$ curve (or the minimum of the u_t curve). Since the maximum of this curve comes at larger values of d at low brightness levels (Figs. 1 and 2) it follows that the best magnification will decrease with decreasing brightness.

The wavy appearance of the lines in Fig. 5 throws some doubt on the theory that the central nervous system "straightens" lines or borders which should otherwise show traces of an irregular cone mosaic. The fact that the irregular nature of these lines can be detected introduces a very important factor into the theory of visual acuity which can best be described as spacial discrimination. If, as a first approximation, it is assumed that visual acuity is essentially a process of retinal illumination discrimination, it is difficult to explain certain visual acuity phenomena such as the classic vernier or straight line offset problem. If one segment of a straight line is displaced an angular distance of only 0.05 minute of arc with respect to the rest of the line, an observer can detect the offset. This situation appears similar to the recognition of the irregularities of the lines described above in which the "amplitude" $\delta\alpha$ of the irregularities is probably about 0.15 minute. If it is possible to detect a displacement this small on a rapidly shifting line segment only 20 minutes in length, and which is seen against a rather confusing background of similar lines, it seems plausible that a displacement of about one-third this amount in a single line several degrees long should be detected, even though the border gradient in this instance may extend over two rows of cones. For line segments at least 2 minutes in width, calculations for the vernier show that the illumination on a row of cones in the gradient on one edge of one line segment differs by about 5 percent from the illumination on that same row prolonged into the other line segment when the relative displacement of the segments is 0.05 minute. Calculations of this sort led Hartridge[7] to believe that spacial discrimination, or "sense of position" as he referred to it, could be explained in terms of retinal illumination discrimination. From the irregular appearance of the lines in the double-slit experiment it is doubtful whether the explanation is quite as simple as this. Probably what one interprets as the borders of a line are the contours formed by cones which receive approximately equal amounts of light, in which case the recognition of the vernier offset and the recognition of the irregularities in the lines of the double-slit diffraction pattern are each a joint phenomenon of both spacial and retinal illumination discrimination. If the images of the line segments on each side of the vernier offset extend over a large number of cones, then the mean position of corresponding rows of cones in the images of the two segments which receive approximately equal amounts of light will be displaced by an amount equal to the offset.

THE RETINAL BASIS OF PERCEPTION ACUITY

Ability of the eye to see individual objects, such as a black disk or line on a white background, is much greater than for resolving detail

[6] W. H. Stiles and B. H. Crawford, Proc. Roy. Soc. London **B112**, 428–450 (1933).

[7] H. Hartridge, J. Physiol. **57**, 52–67 (1923).

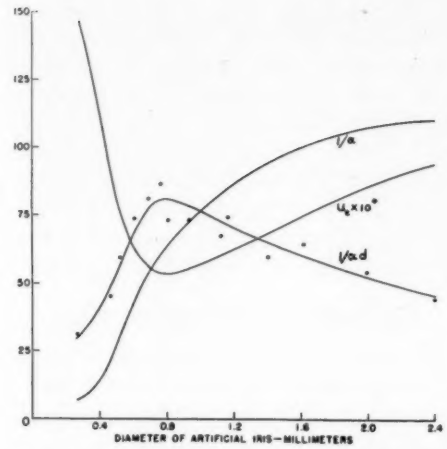

FIG. 6. The curves $1/\alpha d$, u_t, and $1/\alpha$ for a black line are shown as a function of the artificial iris aperture d. To give all curves approximately the same vertical range, u_t is multiplied by a factor of 10^4.

in a more complex object such as recognizing the individual lines in the grating test object. The analysis in Part I of this paper indicated that this difference was largely due to the difference in the distribution of light in the images of the two kinds of objects rather than any essential difference in the response of the retina to the two kinds of objects. This conclusion is borne out by a further investigation of the retinal factors involved in perception acuity. The visual acuity for a black line on a white background has been studied by several investigators, and since the retinal contrast u can be easily determined for this object, the image of a black line is perhaps the most suitable for determining the retinal factors in perception acuity.

Figure 6 shows the results of a visual acuity experiment* in which a black line was observed against a background the brightness of which was 5000 millilamberts. The retinal threshold contrast u_t, the visual acuity $1/\alpha$, and the visual acuity per unit of iris aperture $1/\alpha d$, are all plotted as a function of the artificial iris aperture d. The u_t curve is multiplied by a factor of 10^4 in order to bring all three curves to approximately the same position. The general form of these curves resembles that of the curves for the grating test object (Fig. 3). The corresponding threshold retinal image curves which give the distribution of illumination in the images of the line are shown in Fig. 7 for $d=2.5$ mm and $d=0.8$ mm. The broken curve is a supra-threshold curve (with $d=0.8$ mm) for which the angular diameter of the line is twice its threshold value.

Hecht and Mintz[8] suggested two possible theories to explain the high visual acuity values which they obtained for individual black lines viewed through a 3.0-mm aperture. One of these theories requires that the illumination differential between two adjacent rows of cones be approximately 0.01, or about the same as $\Delta I/I$ for a bipartite field. The alternative theory requires small eye movements to produce about this same differential in a short interval of time in a single row of cones which would move in and out of the blurred image. Either of these theories seems adequate for explaining the resolution of individual lines for normal iris apertures, because in this special case the limit of resolution appears to be determined by retinal illumination discrimination. However, neither theory can account for the peculiar behavior of the u_t or the $1/\alpha d$ curves. From Figs. 6 and 7 it can be seen that u_t is less than 0.006 when $d=0.8$ mm,

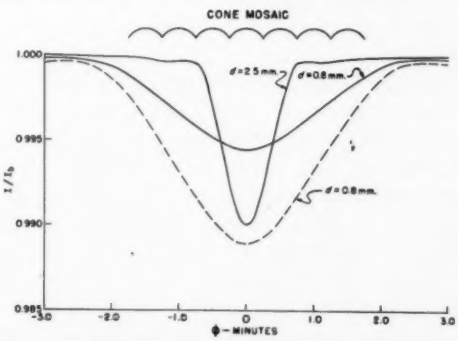

FIG. 7. The solid curves show the distribution of illumination in the image of a black line at threshold for $d=2.5$ mm and $d=0.8$ mm. The broken curve is a supra-threshold curve (with $d=0.8$ mm) for which the angular diameter of the line is twice its threshold value.

* The maximum visual acuity values obtained in this experiment for an aperture of 2.43 mm are considerably greater than those given in Part I of this paper for a black line observed through a similar aperture. This difference was due to the larger background (angular diameter about 1°) used in the second experiment, and to longer searching periods in attempting to resolve the line.

[8] Selig Hecht and Esther U. Mintz, J. Gen. Physiol. **22**, 593–612 (1939).

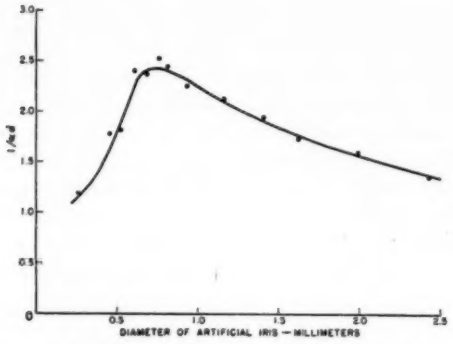

FIG. 8. The $1/\alpha d$ curve for a black disk viewed on a white background with a brightness of 1200 millilamberts.

and that the maximum illumination differential (for this same aperture) between any two adjacent rows of cones is only 0.002. It is also doubtful whether eye movements could account for the differential of 0.006 in a blurred image 5 minutes wide.

In Fig. 8 is shown the $1/\alpha d$ curve for a black disk observed on a white background with a brightness of 1200 millilamberts. The trend of this curve substantiates the findings, reported in Part I of this paper, that retinal illumination discrimination is not the only factor involved in the resolution of a disk.

The initial decrease in u_t, which accompanies a decrease in the aperture d for most test objects, might be explained by assuming that the extension of the illumination gradients over several rows of cones increases any summation effect which may exist between these receptors. However, this assumption introduces more difficulties than it is worth; for example, if extending the gradients over several rows of cones increases summation for test objects such as the line, disk, and grating, then it should do the same for the bipartite field. Apparently this does not happen; u_t (or $\Delta I/I$ as it is often written) remains nearly constant when a bipartite field is viewed through apertures above 1.2 mm, but increases slowly for apertures below 1.2 mm. The concept of insulated pathways is much more satisfactory than any concept requiring neural interaction* not only for explaining the data of visual acuity, but for predicting other visual phenomena as well.

It thus appears that, although visual resolving power can be explained in terms of retinal illumination discrimination under special conditions for certain test objects, the general problem of visual resolving power involves other factors as well. These factors can be determined by considering the energy changes which occur in photo-reception.

THE PROCESS OF PHOTO-RECEPTION

The means by which the energy in a light stimulus is converted into a visual sensation is exceedingly complex but a comparatively simplified scheme of the process explains much of the data of visual acuity. The receptors convert the energy of light into energy of such a form, probably electrical in nature,[9,10] that the nerves can transmit it to the visual cortex of the brain. Investigators are not in agreement as to how much of the total visual function is accomplished in the different parts of the system. Hecht places most of the emphasis on the retinal receptors and the agreement of his theory of photo-reception[11] with experimental data seems to justify this point of view.

The theory of photo-reception used here is Hecht's theory modified by the introduction of the concept of energy conversion rates. The use of this concept makes it possible to deal more effectively with adaptation phenomena and their effect on visual acuity. It will be assumed that the process of photo-reception consists of the following reactions: (a) a photochemical reaction in the receptor in which light is absorbed by a photosensitive substance S which is converted into active photo-products; this part of the process changes the energy of light into chemical or potential energy; (b) a secondary reaction in which the active photo-products are converted to inactive products, and it will be assumed that the energy change occurring in this part of the process ultimately produces a visual sensation; (c) a thermal reaction which utilizes the inactive

* It will be shown in a later paper that an apparent summation does exist between the receptors. However, the quantum theory shows that summation has a statistical rather than a neural origin, so does not conflict in any way with the concept of insulated pathways.

[9] H. K. Hartline, J. Opt. Soc. Am. **30**, 239–247 (1940).
[10] S. H. Bartley, *Vision* (D. Van Nostrand Company, Inc., New York, 1941).
[11] Selig Hecht, Physiol. Rev. **17**, 239–290 (1937).

products to maintain the supply of the photosensitive material S.

If the total initial concentration of S is a, if the concentrations of the active and inactive photo-products at time t are y and x, respectively, the velocities of the reactions (a), (b), and (c) are assumed to be

$$\left(\frac{dx}{dt}\right)_1 = k_1 I(a-x-y)^m, \tag{7}$$

$$\frac{dy}{dt} = -k_3 y^l, \tag{8}$$

and

$$\left(\frac{dx}{dt}\right)_2 = -k_2 x^n, \tag{9}$$

where m, l, and n are the respective orders of the reactions, I the retinal illumination, and k_1, k_2, and k_3 are velocity constants. The value of k_1 will depend also on the units in which I is measured. The visual sensation is assumed to be determined by the velocity of reaction (b), and since vision ceases in only a fraction of a second after the illumination is cut off, the velocity constant k_3 must be very large compared with k_2. If this is true, then y can have only a negligible effect on $(dx/dt)_1$ in Eq. (7) because it will always be very small compared with either x or a. Adding Eqs. (7) and (9) gives for the net rate of formation of the inactive products (neglecting y)

$$\frac{dx}{dt} = k_1 I(a-x)^m - k_2 x^n. \tag{10}$$

If the test object brightness remains constant, dx/dt ultimately approaches zero and the equilibrium condition is therefore

$$KI = x^n/(a-x)^m, \tag{11}$$

which is Hecht's equation of the photochemical steady state. The constant K is the ratio k_1/k_2.

The mathematical description of the second part (b) of the process of photo-reception will be made in terms of an energy equation and differs considerably from Hecht's treatment. The rate dE/dt at which energy is released when the active photo-products are changed to inactive products is assumed to be directly proportional to $-dy/dt$. Because of the large value of k_3, dy/dt will be equal, or nearly equal, to $-(dx/dt)_1$, even when I is not constant; hence dE/dt is directly proportional to $(dx/dt)_1$. If for brevity dE/dt is written as r, and if y is neglected in Eq. (7), then

$$r = CKI(a-x)^m, \tag{12}$$

where C is a constant. If the concentration of inactive photo-products is written as a fraction p of the initial concentration of the photosensitive material S, then $p=x/a$, and Eqs. (11) and (12) can be written in the simplified forms

$$KI = p^n/(1-p)^m \tag{13}$$

and

$$r = r_a KI(1-p)^m, \tag{14}$$

where the constant K is now $(k_1/k_2)a^{m-n}$, and r_a is the limiting value which r assumes in the steady state if the retinal illumination I is increased without limit. If the retinal illumination is increased without limit, $p \to 1$ and $r \to r_a$. It will be assumed that the initial photochemical reaction (a) is unimolecular, $m=1$, and that the thermal reaction (c) is bimolecular, $n=2$.

The equation for the differential threshold can be derived from Eqs. (13) and (14). Suppose that an observer has his attention fixed on the boundary between two adjacent areas for which the retinal illuminations on either side of the image of the border are I_1 and I_2. Probably small rapid eye movements* keep shifting the position of the image of the border before appreciable changes can occur in the concentration of photosensitive material. If $\Delta\phi$ is the mean amplitude of the eye movements, then the concentration of photosensitive material should be nearly constant over an angular distance $\pm\Delta\phi$ on each side of the border, and the steady state equation in the vicinity of the border will be

$$K(I_1+I_2)/2 = p^n/(1-p)^m. \tag{15}$$

The energy rates for the receptors within a distance $\pm\Delta\phi$ of the border will be

$$r_1 = r_a KI_1(1-p)^m = 2r_a p^n I_1/(I_1+I_2), \tag{16}$$

and

$$r_2 = r_a KI_2(1-p)^m = 2r_a p^n I_2/(I_1+I_2). \tag{17}$$

* These eye movements cannot be the result of continuous oscillatory motions or the image would be hopelessly blurred. They are more likely the result of sudden jerks, with small but more or less random amplitudes, which suddenly impress the image of a border on a fresh retinal area.

It will be assumed that the threshold difference in the energy rates $(r_1-r_2)_t$ when the border is just visible is given by the equation*

$$(r_1-r_2)_t=A+B(r_1+r_2)^{\frac{1}{2}}, \tag{18}$$

where A and B are constants. Eliminating r_1 and r_2 between Eqs. (16), (17), and (18), and writing $(I_1-I_2)/I_1$ as u_t gives

$$2u_t/(2-u_t)=c_1/p^n+c_2/p^{n/2}, \tag{19}$$

where c_1 and c_2 are constants. If r_1 and r_2 were not fluctuating quantities, the term $c_2/p^{n/2}$ would drop out of Eq. (19) and the right member of this equation would be identical with that of the differential threshold equation of Hecht.[11] However, with the exception of vision at very low intensities, the fluctuation term is probably the dominating one. If the term c_1/p^n is neglected and if u_t is small then Eq. (19) can be written in the approximate form

$$u_t=c_2/p^{n/2}, \tag{20}$$

which, with $m=1$ and $n=2$, is in good agreement with the differential threshold data of several investigators except for low retinal illuminations near the absolute threshold.

The existence of the so-called "negative after-sensation" can be predicted from Eqs. (13) and (14). Suppose that an observer has had his attention directed for some time on area A of brightness H_1 for which the corresponding retinal illumination is I_1. The steady state equation for the receptors in the image of this area is

$$KI_1=p_1^2/(1-p_1), \tag{21}$$

where p_1 is the concentration of photo-products. If the area is suddenly replaced by a screen of brightness H which results in a retinal illumination I, the initial energy rate r_1 is

$$r_1=r_aKI(1-p_1). \tag{22}$$

Ultimately a new steady state will be established and r_1 will approach a new value r which is given by the equation

$$r=r_aKI(1-p), \tag{23}$$

where p is the concentration of photo-products in the new steady state, the equation for which is

$$KI=p^2/(1-p). \tag{24}$$

If I' is the retinal illumination which will give an energy rate r_1 when the concentration of photo-products is p, then

$$r_1=r_aKI'(1-p). \tag{25}$$

The quantity I' may be regarded as the apparent retinal illumination at the instant the illumination I replaces the illumination I_1 to which the receptors have been adapted. Eliminating r_1 in Eqs. (22) and (25) gives for I'

$$I'=(I^2/I_1)(p_1/p)^2. \tag{26}$$

By solving the steady state equations (21) and (24) for p_1 and p it is possible to express I' as a function of I and I_1 only.

If the adapting area consists of two adjacent areas A_1 and A_2 the brightnesses of which are H_1 and H_2, then at the instant these areas are replaced by the screen of brightness H the apparent retinal illumination I'' for the image of the area A_2 can be determined in the same way as I'. The ratio I'/I'' for the two areas corresponding to A_1 and A_2 will then be

$$I'/I''=(I_2/I_1)(p_1/p_2)^2. \tag{27}$$

This phenomenon has been known classically as the "negative after-sensation" and is apparently the result of unequal concentrations of photo-sensitive material in neighboring retinal areas. At high retinal illuminations Eq. (27) reduces to the equation $I'/I''=I_2/I_1$. At very low brightness levels I'/I'' approaches unity and the negative after-sensation should not be visible if I_1, I_2, and I are all small. If observations are made at progressively decreasing brightness levels, the negative after-sensation should disappear in the fovea before it does in the outer retina because the constant K is much larger for the rods (and perhaps also for the parafoveal cones) than for the foveal cones.

Equation (27) gives the ratio I'/I'' only at the instant the observer views the screen of uniform brightness. The system immediately approaches a new steady state and the ratio I'/I'' approaches unity. However, traces of the negative after-sensation are often visible for at least 10 seconds.

* The derivation of this equation will be given in a later paper. It is based on the assumption that the number of quanta used by the receptors, even at high retinal illuminations, is comparatively small, thus causing r to be a fluctuating quantity.

LOCAL ADAPTATION AND VISUAL ACUITY

Adaptation is the mechanism which enables the eye to function over a large range of retinal illumination. There is some disagreement as to whether adaptation is essentially a photochemical process or a combination of both photochemical and neural processes. It will be assumed here that it is photochemical, although neural factors are doubtless involved. The negative after-sensations are due to local variations in adaptation and have an important effect on visual acuity. Under certain conditions they can raise the threshold contrast, or differential

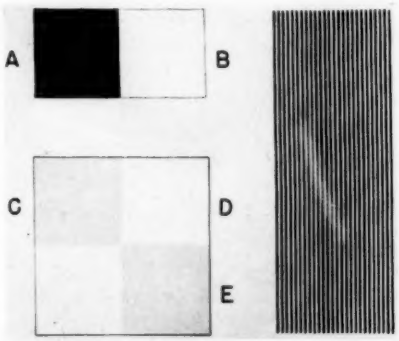

FIG. 9. The border between C and D is plainly visible. If one holds the page at a distance of about 14 inches from the eyes and fixates for about 20 seconds on the small black dot at the center of the parallel lines and then fixates on the border, it will be difficult to see for a few seconds. The border between D and E is not appreciably affected. If one fixates on the border between A and B for about 20 seconds and then on the border between C and D, the relative brightness of C and D will appear reversed.

threshold, to a value which is several times as great as that for a uniformly adapted retina.

The effects of negative after-sensations on visual acuity can be most easily visualized in terms of the "photochemical" image in which the energy rate r is plotted for different parts of the image of a test object. At low intensities, curves plotted in terms of r resemble the retinal image curves plotted in terms of I, but at high intensities the two curves are quite different. If the adaptation β is quantitatively defined by the equation $\beta=1-p$, then the energy rate r in the photochemical image will be

$$r=r_aKI\beta. \qquad (28)$$

If there are large and frequent variations in β over the retina, that is, if $\partial\beta/\partial\phi$ frequently changes from large positive values to large negative values, there will be corresponding changes in $\partial r/\partial\phi$ and the photochemical image will contain spurious gradients or "ripples" which should greatly increase the differential threshold. This effect can be verified by a simple experiment. In Fig. 9 one can easily see the border between areas C and D. If one keeps his attention rigidly fixed on the black dot at the center of the series of parallel lines to the right for at least 20 seconds and then attempts to see the border between C and D, he will find this rather difficult to do. For several seconds the spurious gradients in the negative after-sensation mask the slight gradient between C and D. As one would expect, the appearance of the border between D and E, which is perpendicular to the spurious gradients, remains almost unchanged. Under good laboratory conditions the differential threshold may be as small as 0.01 at a high retinal illumination, but by inducing local variations in adaptation it may be made as large as 0.08 at the same illumination. In photometry it is usually assumed to be about 0.02, but in outdoor vision where the eye is constantly viewing scenes which contain large variations in brightness, a more suitable figure probably would be 0.03 or 0.04.

Figure 10 shows the steady state value of r and the corresponding adaptation β plotted as a function of KI (on logarithmic scales). From Steinhardt's data[12] the value of $1/K$ is about 0.25 lumen per square meter for the foveal cones. When H is in millilamberts and I is in lumens per square meter, the relation between H and I for the reduced eye is approximately

$$I=\rho(a/F)^2(10H),$$

where ρ is the transmission factor (about 50 percent) of the ocular system, a the radius of the artificial iris, and F the focal length* of the eye. Because of the effect observed by Stiles and

[12] Jacinto Steinhardt, J. Gen. Physiol. **20**, 185–209 (1936).

* The fraction a/F is written as an approximation of the quantity $na/(a^2+n^2F^2)^{\frac{1}{2}}$, where n is the index of refraction of the fluids in the eye. Flux calculations in different optical systems are discussed in considerable detail by A. C. Hardy, J. Opt. Soc. Am. **33**, 71–74 (1943).

Crawford[6] this equation should not be used for values of a greater than 1.5 mm. If a is 1.21 mm, and F is 16 mm, then $I=0.029\ H$ approximately. Figure 10 shows that r (or r/r_a) is nearly constant when KI is greater than 10, or when the retinal illumination exceeds 2.5 lumens per square meter. This corresponds with a test object brightness of about 85 millilamberts for an iris radius of 1.21 mm. At very low retinal illuminations r is directly proportional to I for the photochemical steady state.

The constancy of r at high retinal illuminations for the steady state may be the reason why it is so difficult to estimate brightness out of doors. However, if r is constant at high retinal illuminations, it is difficult to see how the recognition of detail can take place because all parts of the visual field should appear equally bright. Apparently the explanation lies in the nature of small, rapid eye movements[5] which keep adaptation constant across the image of a border separating two adjacent areas differing in brightness. If these movements were lacking the adaptation would change abruptly at the border and there would have to be a different steady state equation on each side of the border. Consider the retinal image of two adjacent areas A_1 and A_2 (Fig. 11, curve A) for which the retinal illuminations are I_1 and I_2. If it is

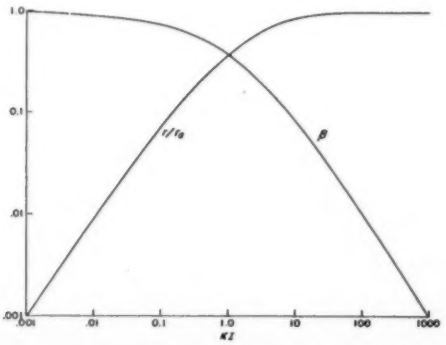

FIG. 10. The energy rate r for the steady state and the corresponding adaptation β are shown as a function of KI.

assumed that the normal flutter of the eyes keeps adaptation, or the concentration of the photosensitive substance S, approximately constant for a small angular distance $\pm\Delta\phi$ on each side of the image of the border, there will be only one steady state equation at the border which is

$$K(I_1+I_2)/2=p^2/(1-p). \tag{29}$$

The energy rate equations on each side of the border will be

$$r_1=r_aKI_1(1-p) \tag{30}$$

and

$$r_2=r_aKI_2(1-p). \tag{31}$$

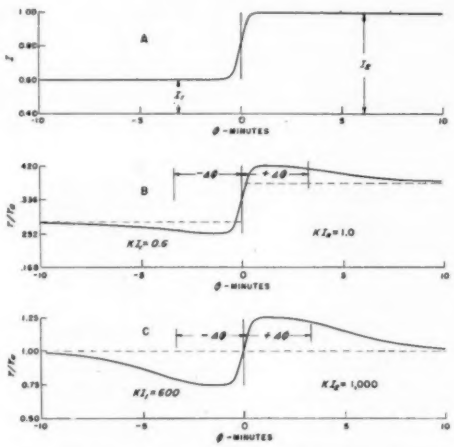

FIG. 11. Curve A shows the form of the retinal image of the border of a bipartite field for which the ratio I_1/I_2 is 0.6. Curve B gives the corresponding energy rates in the photochemical image when I_1 and I_2 are 0.15 and 0.25 lumen per square meter, respectively; C is a similar curve for which I_1 and I_2 are 150 and 250 lumens per square meter.

Dividing (30) by (31) gives $r_1/r_2=I_1/I_2$ which may be written in the differential form

$$\Delta r/r=\Delta I/I=u_i, \tag{32}$$

hence the contrast in the photochemical image is equal to that in the retinal image. It would thus appear that the criterion of energy rate discrimination reduces to that of retinal illumination discrimination provided that the concentration of photosensitive material is constant in the receptors in the vicinity of a border. At a considerable distance from the border the concentration of photosensitive material is not the same and it can be shown that the ratio r_1/r_2 is

$$r_1/r_2=I_1(1-p_1)/I_2(1-p_2). \tag{33}$$

Figure 11, curve B, shows values of r/r_a in the photochemical image of a bipartite field for which I_1 and I_2 are 0.15 and 0.25 lumen per square meter, respectively; in curve C, I_1 and I_2

are 150 and 250 lumens per square meter. These curves indicate that in the image of two adjacent areas differing in brightness, the darker area should appear darkest near the boundary and the lighter area lightest near the boundary. This phenomenon has long been recognized and is sometimes called "brightness contrast." Marshall and Talbot[5] explain "brightness contrast" by using an electro-physiological concept, the recovery cycle, but this seems unnecessary if the explanation can be made in terms of more basic concepts at the receptor level. At low retinal illuminations the form of the photochemical image approaches that of the retinal image (curve A).

It is difficult to believe that vision under the most favorable conditions, in bright light, is almost entirely a border or contour phenomenon, although the experiment illustrated in Fig. 12 bears out this conclusion. Nevertheless, curve C in Fig. 11 is likely to be misleading. Some time is required before the photochemical steady state can be considered established, even with steady fixation, and in the normal habits of vision, the direction of one's attention is being constantly shifted through rather large angles so that in visual tasks not requiring steady fixation, there is probably a closer correspondence between r and I at a considerable distance from a border than Eq. (33) or curve C would indicate. However, visual acuity does not seem to be impaired under steady fixation because the small, rapid eye movements, which keep adaptation constant in the vicinity of a border, are involuntary. It is possible that the tendency of r to approach a stationary value (and yet to change suddenly in the vicinity of a border) has considerable bearing on the phenomenon of brightness constancy and perhaps even color constancy. If a grey area is placed on a bright white background, the value of r in the image of the area near its border will be much less than when it is on a black background. For this reason a dark grey area with a black background may actually appear brighter than a light grey area on a bright white background.

The experiment illustrated in Fig. 9 seems to explain the effect of background size on visual acuity. It also seems to explain the apparent interference of borders which Fry and Bartley[10] assumed was due to some sort of interaction or the spreading of activity from one part of the retina to another. When a test object such as a very small bipartite field is viewed against a background considerably brighter or darker than

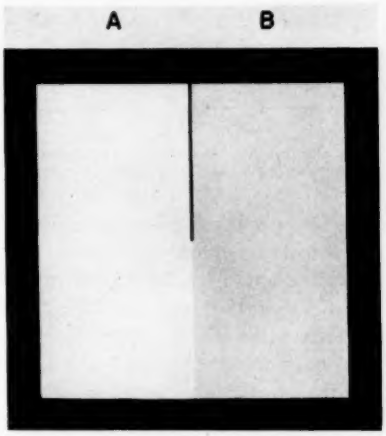

FIG. 12. Area A is obviously brighter than area B. If one places this figure in bright light and covers up the lower half of the figure and fixates on the black line, it is difficult to judge which area is brighter. The line drawn on the border blots out the energy rate gradient shown in Fig. 11 between $-\Delta\phi$ and $+\Delta\phi$. If the eyes are now rapidly shifted from one area to the other, the brightness difference once more becomes noticeable.

the test object, it is not possible for the observer to keep from occasionally directing his attention to the border between the object and its background; for a small bipartite field 20 minutes in angular diameter, this may happen at least once a second. The negative after-sensations resulting from the involuntary glances at the sharp boundary will increase the threshold contrast u_t. This seems to be most clearly shown in the data of Steinhardt[12] for bipartite fields of different angular diameters. As the fields become larger the effect drops off rapidly, perhaps indicating that for larger fields the observer glances at the boundary less often. Even so, the after-sensation persists for several seconds and the effect becomes negligible only for rather large fields (5° or more in angular diameter). It is possible that there is some "interference" between borders but for foveal vision at high brightness levels its effect is probably very small compared with the effect

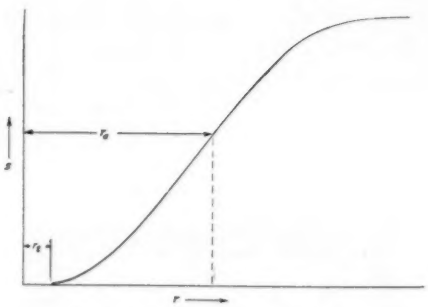

FIG. 13. Possible relation between the visual sensation s and the energy rate r if the adaptation β is constant.

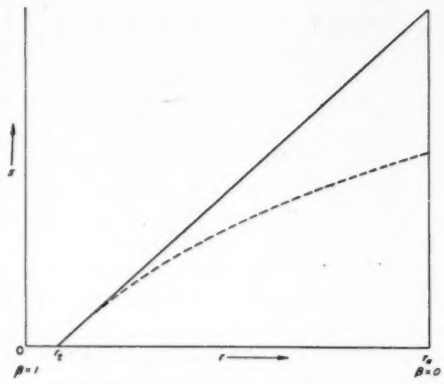

FIG. 14. Possible relation between the visual sensation s and the energy rate r for the photochemical steady state. The adaptation β varies from 0 to 1.

produced by negative after-sensations.* The effect of perceiving a border of small contrast after viewing another of large contrast can be seen in an exaggerated way from Fig. 9. If one keeps his attention rigidly fixed for about 15 seconds on the border between A and B and then fixates on the border between C and D, the area C will appear brighter than D for several seconds. By varying the brightness of C and D until they appear equally bright, this experiment could be modified to estimate the relative adaptation in the two areas at any instant after fixation on the boundary between A and B.

Visual sensations** cannot be measured, but occasionally it is helpful to assume that if the adaptation β (or the concentration $1-p$ of photosensitive material) is constant, the visual sensation is related to the energy rate r by a curve (Fig. 13) which resembles the characteristic curve of a photographic film. If β is constant, the retinal illumination I is directly proportional to r, hence if a scene containing objects of different brightness is suddenly imaged on the retina, there will be a direct correspondence between the retinal, photochemical, and cortical images provided that the largest and smallest values of r fall on the linear portion of the sensation curve. If there is steady fixation on some part of a scene, β will not be constant over the retina but the contrast in all three images should be the same because β is constant in the vicinity of borders.

If r were a non-fluctuating quantity it is probable that there would be a constant sensation interval Δs_0 corresponding to the threshold difference Δr_t in the energy rates for the receptors in two adjacent retinal areas, the difference in the illumination of which could just be discriminated. If Δs_0 were constant, then in the linear part of the sensation curve Δr_t would also be constant. However, it is probable that r is subject to physical fluctuations, in which case it would be difficult to relate Δs_0 and Δr_t.

Figure 14 shows a possible relation between s and r in the steady state condition for the whole range of adaptation in which β goes from 0 to 1. The dotted line shows a possible relation between s and r if neural factors are involved in adaptation. In these curves r can never exceed r_a, since r_a is defined as the maximum value which r can attain if the system is in equilibrium.

THE BLURRING OF RETINAL IMAGES

If one observes a test object through an aperture of about 0.5 mm, the borders of the test object are noticeably blurred even at threshold. This is not surprising when one considers

* The effect of negative after-sensations on differential thresholds and visual acuity should not be confused with the area-threshold effect. This latter phenomenon has an entirely different origin and is apparently caused by fluctuations in the number of light quanta which the receptors utilize.

** It is assumed that the visual sensation is some sort of disturbance involving an energy change in the visual cortex of the brain. It is assumed that the magnitude of this disturbance is determined by the energy rate r. The word "sensation" as used by psychologists may not have quite this shade of meaning. The representation of the relative magnitude of a sensation as shown in Fig. 12 is not compatible with Fechner's law. However, so little is known about the nature of sensations that any attempt at their quantitative evaluation must still be regarded as speculation.

the distribution of light in retinal images, but more puzzling is the fact that the borders appear sharp when viewed through a large aperture even though the retinal image is blurred just as it is for small apertures, but to a less degree. Neurophysiologists have accounted for the sharp appearance of contours by assuming that a neural mechanism operating in the post-retinal system "sharpens" the blur, so that the cortical image is a more truthful replica of the test object than the retinal image. The experiments for determining the data for the u_t and $1/\alpha d$ curves seemed to throw some light on this hypothesis. It was found that when the test objects were viewed through small circular apertures there was no definite aperture at which the border became sharp; the blurring appeared to decrease with increasing values of d, becoming almost imperceptible when d was 1.5 mm. For the larger values of b, the double slit formed a diffraction pattern with such steep gradients that the lines seemed to have an "unnatural" sharpness.

The factors involved in blurring are illustrated in Fig. 15 in which the photochemical image of a black line on a bright white background is plotted for an artificial iris aperture of about 0.8 mm. It is assumed that a threshold difference Δr_t must exist between the energy rates for the receptors A and B in the center and edge of the image before discrimination can take place. For the intervening receptors C, Δr will be less than Δr_t and it is assumed that the C receptors are not involved in the discrimination process.* Apparently no neural mechanism is necessary to account for the sensation of sharpness or discontinuity. If the rows of A and B receptors are separated by several rows of the neutral C receptors, the image will appear blurred. If the number of rows of C receptors is decreased, blurring will be diminished and will vanish (or be reduced to a minimum) when the A and B rows of receptors are adjacent.

* From the standpoint of the quantum theory this assumption is not strictly valid. If r is a fluctuating quantity, one can only state that the probability that at any given instant the C receptors will be involved in the discrimination process is less than the probability that the A (or B) receptors will be involved. The quantum theory also shows that regardless of how "sharp" the contours in the retinal image are, they should appear blurred in very faint light.

Figure 15 seems to explain the rise in the u_t curves and the fall in the $1/\alpha d$ curves which occurs when the diameter of a small artificial iris is decreased. It has been shown that if small eye movements keep adaptation constant over an angular distance $\pm\Delta\phi$ on each side of a border, then $\Delta r/r=\Delta I/I=u_t$. However, if the aperture is very small, the gradients in the retinal image will extend over an angular distance which exceeds $\Delta\phi$. Adaptation will not be the same throughout the gradient and $\Delta r/r$ will be less than $\Delta I/I$ or u_t. For this reason u_t must increase in order that $\Delta r/r$ remain constant. The initial fall in the u_t curves which occurs when the diameter of moderate apertures is decreased is an area-threshold phenomenon and its explanation requires the use of fluctuation theory.

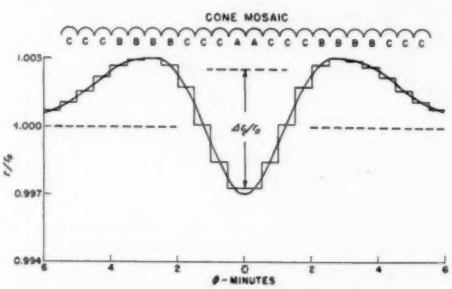

FIG. 15. The photochemical image of a black line viewed on a bright white background. The broken line shows the probable effect of the retinal mosaic on the energy rate. It is assumed that at threshold only the A and B cones are involved in the process of discrimination.

THE EQUATIONS OF VISUAL ACUITY

In Part I of this paper, visual acuity equations were derived for high brightness levels and it was shown that the contrast u in the retinal image of a simple test object could be expressed by the equation

$$u=vf(\alpha), \tag{34}$$

where v is the contrast between the object and its background, α is the angular diameter (or some other angular measurement) of the test object, and $f(\alpha)$ a function which for most objects can be expressed as an ascending power series in terms of α. For a line object $f(\alpha)$ is

$$f(\alpha)=A\alpha-C\alpha^3+E\alpha^5-G\alpha^7+\cdots, \tag{35}$$

in which the coefficients of the even powers of α are missing. For a disk $f(\alpha)$ is

$$f(\alpha)=B\alpha^2-D\alpha^4+F\alpha^6-H\alpha^8+\cdots, \quad (36)$$

in which the coefficients of the odd powers of α are missing.

Substituting u_t for u in Eq. (34) and eliminating u_t between this equation and the approximate differential threshold Eq. (20) gives

$$1/f(\alpha)=vp^{n/2}/c_2, \quad (37)$$

which is an approximate form of the general visual acuity equation.

For a line of very small angular diameter, powers of α higher than the first may be neglected in Eq. (35) in which case the visual acuity $1/\alpha$ is thus directly proportional to $p^{n/2}$. Assuming that $f(\alpha)=A\alpha$ the visual acuity equation for the line may be written

$$1/\alpha=(1/\alpha_0)vp^{n/2}, \quad (38)$$

where $1/\alpha_0=A/c_2$ and is the visual acuity at high brightness levels when the object background contrast v is unity.

For a disk of very small angular diameter, powers of α higher than the second can be neglected in the series for $f(\alpha)$, and the visual acuity is thus directly proportional to $p^{n/4}$. Assuming that $f(\alpha)=B\alpha^2$, the visual acuity equation for a small disk is

$$1/\alpha=(1/\alpha_0)v^{\frac{1}{2}}p^{n/4}, \quad (39)$$

where $1/a_0=(B/c_2)^{\frac{1}{2}}$ and is the visual acuity at high retinal illuminations when the contrast v is unity.

When written in terms of the solid angle τ subtended by the test object, Eq. (39) becomes

$$1/\tau=(1/\tau_0)vp^{n/2}, \quad (40)$$

which applies to any individual object of any irregular shape which appears to become a formless spot as the threshold is approached. However, this equation has not been verified experimentally and Eq. (39) has not been verified for small values of I. Both Eqs. (38) and (39) are in good agreement (see Part I of this paper) with visual acuity measurements at high retinal illuminations in which the object-background contrast v was varied. For a black line observed on a white background ($v=1$), Eq. (38) is in good agreement with the cone data of Hecht and Mintz[8] except for very low illuminations near the absolute threshold.

When v is unity, Eq. (37) should hold for a complex test object such as the grating, but for this object $f(\alpha)$ is of such a form that the visual acuity equation cannot be reduced to simple equations such as (38) and (39).

In making numerical calculations from the visual acuity equations, the variable p can be determined by solving the steady state equation (13). If $m=1$ and $n=2$, the steady state equation in terms of p is

$$2p=[(KI)^2+4KI]^{\frac{1}{2}}-KI. \quad (41)$$

Theoretically Eqs. (38) and (39) should hold only for values of α sufficiently small that it is permissible to neglect all terms in the power series expansions except the first. Nevertheless, visual acuity measurements for the disk test object show that Eq. (39) holds for rather large values of α, at least up to 2 minutes and perhaps larger. It is probable that Eqs. (38) and (40) also hold for rather large values of α, but this has not been verified experimentally. Retaining only the first term in the power series expansion for $f(\alpha)$ is equivalent to assuming that for any given brightness level the retina responds to a constant flux differential rather than either a constant illumination differential or a constant energy rate differential. If the retina responds to a constant flux differential, then u_t must decrease with increasing area of the image. This area-threshold effect indicates that summation exists among the receptors and its investigation requires the use of the quantum theory.

OTHER THEORIES OF VISUAL RESOLVING POWER

Like many of the other problems of vision, the problem of visual resolving power has been approached in different ways by investigators in several fields of science. Wilcox[13,14] thought that visual acuity could be expressed in terms of the subjective width of test objects, or in terms of "irradiation." If a bright line or bar is viewed against a dark background it will appear wider at high illuminations than at low illuminations; this change in subjective width is known as

[13] W. W. Wilcox, Proc. Nat. Acad. Sci. **18**, 47–56 (1932).
[14] W. W. Wilcox, J. Gen. Psychol. **15**, 405–435 (1936).

irradiation. In the experiments of Wilcox, an observer viewed two parallel bright bars (2.4 minutes in width) against a dark background, which were arranged so that their separation could be varied. The observer adjusted the distance between the bars until the width of the interspace appeared equal to the sum of the widths of the bars. If the bars are subjectively widened the actual width of the interspace would have to exceed somewhat the sum of the actual widths of the bars.

Wilcox shows that if R is the apparent distance between the bars when this distance appears equal to the sum of their apparent widths, then the distance d by which each border of the bar

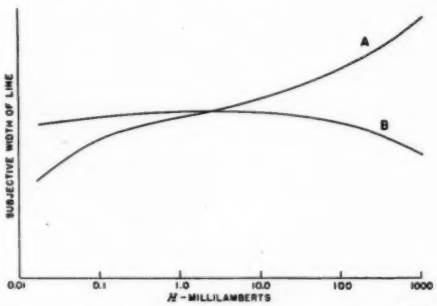

FIG. 16. Curve A is the subjective or apparent width of a white line on a dark background plotted as a function of the brightness of the line; B is a similar curve for a black line on a white background plotted as a function of the background brightness. The actual width of both lines was about 2 minutes.

is displaced by irradiation is

$$d=(R-2x)/4,$$

where x is the actual width of the bar. He found that at high illuminations d decreases with decreasing light intensity until it reaches zero and then increases again at low illuminations. This is precisely the manner in which the visual acuity for two white bars on a dark background varies with illumination (see Fig. 2) and Wilcox therefore assumed that irradiation and visual acuity were closely related.

The apparent widening of narrow bars at low illuminations is a good example of how misleading some experiments in vision and their interpretation can be. The experiment of Wilcox appears straightforward and its interpretation logical, but if one performs the experiment in a more direct manner, an entirely different result is obtained. Consider a narrow white line viewed against a dark grey background and illuminated by two point sources of light, the intensity of which can be varied. A ruler is placed in front of one light so as to cast a sharp shadow on a portion of the line where the illumination will be only that received by the other light. At high illuminations one would expect the portions of the line outside the shadow to appear wider than that part of the line within the shadow and this is what actually happens. From the results of Wilcox's experiment, one would expect at low illuminations that the line would appear wider within the shadow than outside. Just the opposite occurs and the difference in apparent width between the shaded and unshaded parts of the line is even greater at low illuminations than at moderate illuminations. Figure 16, curve A, shows the apparent width of a thin white line on a dark ground plotted as a function of its brightness. Curve B shows the corresponding relation for a thin black line on a white ground. The apparent widths of a black line within and outside the shadow are so nearly equal that it is difficult to detect any difference except at high illuminations. The ordinates of curves A and B indicate only the general relation between relative subjective line width and brightness. The quantitative determination of this relationship would require a more elaborate experiment in which the width of the shadowed line segment could be varied until it matched the width of the unshadowed segment.

In both of the irradiation experiments just described, it is impossible to obtain a stable state of adaptation of the retina when white lines are viewed on a black background, but this difficulty can be partially overcome by using a dark grey background which has a brightness of about one-tenth that of the white line.

The subjective narrowing of a white line on a dark background can be predicted from the energy rate equations. The retinal image of a bright line 2 minutes in width extends over about 8 rows of cones, or even more if chromatic and spherical aberration are appreciable. When the brightness of the line is decreased the energy rate r for the outer rows of cones drops below some threshold value r_i; this results in the

apparent narrowing of the line. The "irradiation" effects which Wilcox observed for two parallel white lines at low brightness levels can be explained in a somewhat similar manner. The photochemical image of two white lines on a black background resembles that of a single black line on a light background (Fig. 15) except that r drops off rapidly at the outer borders of the image. There will be a certain number of rows of active B receptors in the center of the images of the lines and a certain number of rows of active A receptors at the center of the image of the interspace. Between the A and B receptors there will be a number of rows of neutral C receptors in the gradients of the image of the interspace for which the energy rate difference Δr with respect to the A or B receptors will be less than Δr_t. If the size of the image is held constant and if the illumination is decreased the number of rows of the neutral C receptors will increase. Since the size of the image (and hence the total number of rows of receptors) is constant, the increase in the number of rows of C receptors must be made at the expense of the B and A receptors, especially the latter. In order for the width of the interspace to appear equal to the sum of the widths of the two white lines, the separation of the lines must therefore be increased.

The criterion of resolution is arbitrary; two white bars may be considered resolved when the interspace just disappears, or they may be considered resolved when the interspace has some predetermined fraction (or multiple) of the apparent width of either bar. Values of visual acuity obtained by any one criterion should bear fairly consistent ratios to the corresponding values obtained by another criterion, provided that the bars are not too wide. By making the multiple equal to 2, Wilcox was actually measuring visual acuity with a rather coarse criterion and it is not surprising that he found his measurements consistent with those obtained with a criterion for which the multiple was zero.

The poor visual acuity for two white bars at high illuminations on a dark background can be explained in terms of the sensation curve. When only a very small area of the retina is illuminated the concentration of the photosensitive material S remains high over most of the retina, and unless the eyes remain motionless the energy rate r in the image area will be considerably higher than the rate r_a which would prevail in the steady state. If r is sufficiently great to correspond to some point on the "shoulder" of the sensation curve, then Δr_t will have to be comparatively large in order that a minimal sensation interval Δs_0 be maintained. This will result in an increase in u_t and thus require a greater separation of the bars than would be necessary for the adapted retina.

The retinal illumination discrimination theory of visual resolving power was proposed by Helmholtz, and in some form or other, has been the most commonly accepted theory. In some respects the theory developed in this paper is similar to the illumination discrimination theory but differs from it considerably in that the concept of illumination discrimination is replaced by the concept of energy rate discrimination. The use of receptor energy rates rather than the illumination of the receptors makes it possible to deal more effectively with adaptation phenomena and their effect on visual acuity. With the aid of the quantum theory this idea can be applied to area-threshold and time-threshold phenomena.

Marshall and Talbot[5] have recently proposed a theory of visual acuity which is so different from that developed in these papers that it is difficult to compare the applications of the two methods to any specific problem. They use a neuro-physiological approach in which most of the emphasis is on the post-retinal system. According to this point of view, neural mechanisms play an important part and visual acuity and contour recognition are achieved to a large extent in the visual cortex and associated systems. They point out that the grain of the visual cortex is in effect much finer than the grain of the retina and attribute certain accomplishments of the visual system, such as the perception of the vernier offset, to both the fineness of the cortical grain and the operation of certain neural mechanisms. This would mean that the cortical image and the test object would have in common certain energy differentials which would be lacking in the retinal and photochemical images. In view of the physical concept of entropy, it is hard to see how the energy in

the cortical image could be in a more highly organized state, with respect to the test object, than the energy in the retinal image. This would be equivalent to obtaining a sharp enlargement from a blurred photographic negative by manipulating the focusing mechanism of the enlarger so as to obtain an "equal and opposite blur."

Marshall and Talbot apply their theory to explain the irradiation experiments of Wilcox which they interpret in the same manner as he did. This does not agree with the findings reported here which show that the subjective width of a line should decrease with decreasing values of the retinal illumination I for all values of I.

Since the problems of vision have been investigated by workers in different sciences, it is only natural that there should be a number of diverse opinions as to the nature of the processes involved in visual resolution. Some of these ideas are discussed by Walls[15] in a recent paper on human visual resolution which contains a good bibliography (99 references) on this subject.

There is much to be learned yet about the physical and chemical factors involved in visual acuity. The actual process of photo-reception is undoubtedly more complex than the simplified theory used here. Instead of only one active sensitive material S in the cones, there may be two or more such substances in each cone differing widely in initial concentrations and possessing different velocity constants k_1 and k_2. Such an arrangement would enable a receptor to function through a greater intensity range than would be possible with only one substance. A substance with a low initial concentration but with a large value of k_2 and a small value of k_1 would function at high intensities, while a substance with a high initial concentration but with a small value of k_2 and a large value of k_1 would function at low intensities. This latter requirement seems to be met in the rods in which the velocity constant k_2 for the regeneration of visual purple is so small that more than 30 minutes are required for dark adaptation. The existence of two or more photosensitive substances in each receptor would modify to some extent Eqs. (7)–(9) and subsequent equations derived from them.

[15] Gordon L. Walls, J. Opt. Soc. Am. **33**, 487–505 (1943).

SUMMARY

The function of the retina in resolving power acuity and in perception acuity can be determined from visual acuity experiments the results of which can be represented by the u_t curves, or for many test objects, by the simpler $1/\alpha d$ curves. These curves show that factors other than retinal illumination discrimination are involved in visual resolving power, although for certain test objects viewed through a given iris aperture it may be assumed that visual acuity is determined by retinal illumination discrimination. For individual objects such as disks and lines, visual acuity is much higher than for complex objects such as the grating; nevertheless the retinal factors involved in both instances are substantially the same.

The ultimate resolving power of the retina can be determined by means of the double-slit diaphragm. This experiment shows that the resolving power of the retina is about 300 lines per mm, although traces of an image are visible at 500 lines per mm.

The photochemical system of the retina performs a dual function. It converts the energy of light into some other form of energy which can be transmitted to the visual cortex of the brain, and it regulates the amount of light which the receptors utilize. This regulating mechanism is of such a nature that the energy conversion rate is nearly constant at high retinal illuminations for the photochemical steady state. In bright light there is thus a reciprocal relation between retinal illumination and adaptation. However, if the energy rate is constant throughout the retina, the recognition of detail could not take place in bright light unless eye movements keep adaptation constant in the vicinity of borders, in which case the contrast in the photochemical image would equal that in the retinal image.

Negative after-sensations are probably the result of variations in adaptation in different parts of the retina. They have an important effect on visual acuity.

If adaptation is constant, it is convenient to assume that the curve relating the visual sensation s and the energy rate r resembles the characteristic curve of a photographic film.

Irradiation effects such as the low visual acuity for two bright bars on a dark background can be explained in terms of such a curve.

A general equation can be derived which for any given test object expresses the visual acuity in terms of the retinal illumination (or the test object brightness), and the contrast between the object and its background.

By assuming that visual resolving power is determined by energy rate discrimination, rather than by retinal illumination discrimination, it is possible to explain most of the data of visual acuity except area-threshold phenomena. If it is assumed visual stimuli are non-fluctuating, it is not possible to explain the effect of area on absolute and differential thresholds without invoking the concept of neural interaction. However, if the number of light quanta actually utilized by the receptors is comparatively small, the magnitude of visual stimuli will fluctuate about a mean or expected value. By assuming that visual stimuli are subject to physical fluctuations, it is possible to account for area-threshold effects and still retain the concept of insulated pathways. This phase of vision will be discussed in a later paper.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 34, NUMBER 12 DECEMBER, 1944

Visual Resolution as a Function of Intensity and Exposure Time in the Human Fovea

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(Received June 16, 1944)

FACTORS operating in human visual resolution have recently been analyzed in detail by Walls.[1] As he indicates, numerous studies[2] have shown the dependency, in part, of resolution of detail by the eye on the intensity of illumination. The precise relation between visual resolution and illumination may be quantitatively interpreted in terms of retinal intensity discrimination. Thus, Hecht and Mintz[3] have explained the resolution of a wire suspended in front of an illuminated background in terms of slightly less illumination and excitation of one row of sense organs than of receptors on either side. In addition to the separate activity of rods and cones characteristic of other visual functions, their results indicate that, at maximal brightness, the eye can resolve a wire subtending a visual angle of only 0.5 sec. This resolution is about 60 times as fine as that commonly obtained with other test objects, such as gratings and the Landolt broken circle. When the light intensity distribution in the diffracted retinal image is computed, it becomes evident that the limiting factor in maximal resolution for this type of test object is retinal intensity discrimination. For this reason, Hecht and Mintz argue that visual resolution at any intensity may be determined by the capacity of the eye for intensity discrimination. An equation derived from Hecht's theory of intensity discrimination[4] accurately describes the relation obtained between just-resolvable angle and intensity.

In intensity discrimination, the energy of a flash (or product of ΔI, the just-discriminable difference in intensity, and exposure time) is constant for any prevailing intensity. Graham

[1] G. L. Walls, J. Opt. Soc. Am. **33**, 487 (1943).

[2] See, for example, W. Uhthoff, Graefes Archiv. f. Ophthal. **32**, Abt. 1, 171 (1886) and **36**, Abt. 1, 33 (1890); A. König, Sitz. d. Akad. d. Wiss. Berlin **8**, 559 (1897); and more recently, S. Hecht, J. Gen. Physiol. **11**, 255 (1928); R. J. Lythgoe, Great Britain Med. Res. Coun. Spec. Rep. Ser. No. 173 (1932); and S. Shlaer, J. Gen. Physiol. **21**, 165 (1937).

[3] S. Hecht and E. U. Mintz, J. Gen. Physiol. **22**, 593 (1939).

[4] S. Hecht, J. Gen. Physiol. **18**, 767 (1935).

and Kemp[5] have demonstrated the reciprocity between ΔI and exposure time up to a critical duration beyond which intensity discrimination occurs independently of exposure time and depends only on ΔI. The critical duration is a function of the basic adapting intensity[6] and probably indicates the onset of the sensory response which interrupts the action of light on the retina. Hecht's theory readily accounts for these results on the basis that a constant amount of photolysis determines brightness discrimination. For exposure times less than the critical duration, the stimulus duration is a parameter of Hecht's equation for intensity discrimination.

In view of the applicability of Hecht's theory to visual resolution as well as brightness discrimination, we are especially interested in the influence of variation in exposure time upon the relation between visual resolution and intensity of illumination. Previous experiments[7] would lead us to expect a similar relation for visual resolution as for brightness discrimination when the stimulus duration is systematically varied.

The subject fixates the center of a circular black area which is surrounded by a dimly illuminated circular area. The test flash in the form of a narrow bar appears in the center of this configuration and thus stimulates the fovea. The subject resolves the direction (horizontal or vertical) of the narrow illuminated slit. The intensity threshold for foveal resolution is determined for stimuli of different widths and durations. The fixation and adaptation conditions, described in more detail below, are similar to those used in recent studies on the area-intensity relation.[8]

METHOD

The two experimenters alternated as subjects. Each session began with an initial 15-min. period of dark adaptation. Threshold determinations were made by a modified method of limits for each of seven exposure times and 10 different widths of the illuminated slit. Each experimental period lasted approximately 1 hr. 20 min. Before each pair of threshold determinations the subject observed for 15 sec. a rectangular, opal-glass, light-adaptation screen with a brightness of 330 ml. The light adaptation was followed by 105 sec. of dark adaptation. The two minutes of combined light and dark adaptation served to maintain pupillary size and retinal excitability at an approximately even level for each determination.

Following the combined light and dark adaptation the subject fixated the black center of a faintly illuminated foveal-fixation screen. The evenly illuminated surround not only maintained adequate fixation, but also prevented distortion of the theoretical relationship between visual resolution and illumination which might be caused by inadequacy or lack of surround.[9] A stereoscopic hood with the lenses removed and the right eye-opening blacked-out was firmly attached to the bench before the subject, ensuring monocular determinations and aiding adequate fixation. In a threshold determination the experimenter varied the intensity of a beam of light in steps of about 0.07 log unit. A single flash was presented at each brightness value, beginning with a flash that was definitely subthreshold. For each threshold determination the stimulus slit was either vertical or horizontal in accordance with a prearranged random order of direction. The subject responded to all presentations of the flash, either by indicating the direction as vertical or horizontal, or by reporting that he could not determine the direction. A second threshold determination was made for a given slit before the subject was again light-adapted.

The entire procedure, including the combined light and dark adaptation, was repeated for the remaining stimulus slits. All paired determinations were completed within a 3-min. period and approximately 15 sec. were allowed to elapse between presentations of the flash.

An experimental session for a given exposure time consisted of two paired determinations for each of the 10 stimulus slits. The presentation order was varied systematically with regard to

[5] C. H. Graham and E. H. Kemp, J. Gen. Physiol. **21**, 635 (1938).
[6] See also, M. Keller, J. Exper. Psychol. **28**, 407 (1941).
[7] In addition to the above, C. H. Graham and C. Cook, Am. J. Psychol. **49**, 654 (1937).
[8] C. H. Graham, R. H. Brown, and F. A. Mote, Jr., J. Exper. Psychol. **24**, 555 (1939); C. H. Graham and N. R. Bartlett, J. Exper. Psychol. **24**, 574 (1939) and **27**, 149 (1940).
[9] P. W. Cobb, Am. J. Physiol. **36**, 335 (1914); and J. Steinhardt, J. Gen. Physiol. **20**, 185 (1936).

TABLE I. Logarithm of threshold intensity of resolution as a function of width and duration of an illuminated slit.

Logarithm of visual angle in minutes	Logarithm of threshold intensity in millilamberts													
	189 msec.		87.1 msec.		42.3 msec.		20.9 msec.		10.7 msec.		6.27 msec.		4.13 msec.	
	RHB	JIN	RHB	JIN	RHB	JIN	RHB	JIN	RHB	JIN	RHB	JIN	RHB	JIN
−0.889	−0.358	−0.400	−0.040	−0.083	0.265	0.283	0.545	0.450	0.748	0.703	1.148	1.078		
−0.644	−0.575	−0.655	−0.348	−0.355	−0.013	−0.050	0.240	0.300	0.433	0.420	0.880	0.858	1.185	1.230
−0.380	−0.918	−0.910	−0.565	−0.573	−0.228	−0.300	−0.008	−0.018	0.268	0.198	0.630	0.480	0.845	0.883
−0.204	−1.078	−1.113	−0.748	−0.793	−0.425	−0.413	−0.158	−0.155	0.080	−0.003	0.500	0.380	0.680	0.760
−0.115	−1.250	−1.298	−0.930	−1.005	−0.535	−0.588	−0.328	−0.178	−0.058	−0.123	0.270	0.235	0.603	0.625
0.019	−1.455	−1.403	−1.078	−1.175	−0.733	−0.703	−0.485	−0.405	−0.203	−0.230	0.145	0.120	0.503	0.455
0.096	−1.445	−1.545	−1.238	−1.173	−0.795	−0.780	−0.620	−0.463	−0.348	−0.323	0.043	0.063	0.285	0.425
0.224	−1.538	−1.663	−1.380	−1.275	−0.923	−0.935	−0.720	−0.558	−0.438	−0.415	−0.060	−0.050	0.155	0.265
0.442	−1.668	−1.773	−1.513	−1.465	−1.140	−1.170	−0.865	−0.760	−0.553	−0.585	−0.240	−0.198	0.048	0.055
0.704	−1.988	−2.063	−1.745	−1.755	−1.423	−1.410	−1.048	−1.110	−0.783	−0.838	−0.478	−0.398	−0.185	−0.108

both width of stimulus and exposure time. This procedure permitted the detection and control of any practice effect that might develop and also of any shift in the threshold criterion *per se*. The vertical or horizontal direction of the rectangular stimulus was also varied for each presentation in accordance with a predetermined order so that, in each group of four determinations for a given stimulus area and a given exposure time, two were for the vertical direction and two for the horizontal direction. A control was thus secured for both guessing at the direction and any undesirable effects caused by astigmatism.

Since two threshold determinations were made for each stimulus width during each of four series with a given exposure time, eight determinations were made for each subject for each slit at a given exposure time. The threshold of resolution has been computed by averaging the eight readings available for each exposure time and slit. This number was considered sufficient in view of the previously determined consistency of each observer in making similar judgments.

The apparatus used was essentially that of an earlier experiment.[10] It involves an optical system for presenting a stimulus of uniform light distribution and variable area, intensity, and duration; a foveal-fixation screen; and a light adaptation box. The optical system comprises a lamp, neutral-tint filters, and wedge for the control of intensity, a constant speed motor, and slotted disks for varying the exposure time, and a metal stimulus holder.

Light from a 1000-w Mazda projection lamp at 110-v d.c. passes through a calibrated Wratten neutral-tint wedge and its associated balancing wedge. Lenses focus the light on a slit, 2.54 mm in width. A camera shutter is used with a motor and disks, rotating near the slit, as a selective device for the presentation of a single flash when desired by the experimenter. Several disks are used to give flashes of varying duration. Radial slots of different angular widths are cut from the periphery of each disk. Since the disk revolves at a constant speed, the width of the slot determines the duration of the flash presented by a given disk.

The intensity of the stimulus flash is controlled by the use of Wratten neutral-tint filters in combination with a Wratten neutral-tint wedge. Light passes through the filters to the opal-glass stimulus surface. Between the opal-glass and the subject there is inserted a metal plate with a circular aperture of 40.8 mm (visual angle of 28.9 min.), which restricts the length of the illuminated stimulus slit. The stimulus slit is further delimited by specially prepared photographic plates in which the width of an unexposed slit in the center of each plate determines the width of the stimulus area.

The foveal-fixation screen consists of an illuminated white cardboard in front of the opal-glass stimulus surface. The stimulus flash appears in a circular opening in the center of the cardboard. Shielded flashlight bulbs illuminate the foveal-fixation screen at a brightness of 0.01 ml. All observations are for the fovea of the left eye. The subject views the stimulus field through the stereoscopic hood which is 486 cm from the stimulus field and centers the subject's left eye in the extended optical path.

[10] R. H. Brown and J. I. Niven, J. Exper. Psychol. **34**, (1944). The methods and results of the present paper are described in full detail in the doctoral dissertation, "The influence of stimulus duration upon the relation between visual resolution and illumination," by J. I. Niven, on file in the Clark University Library.

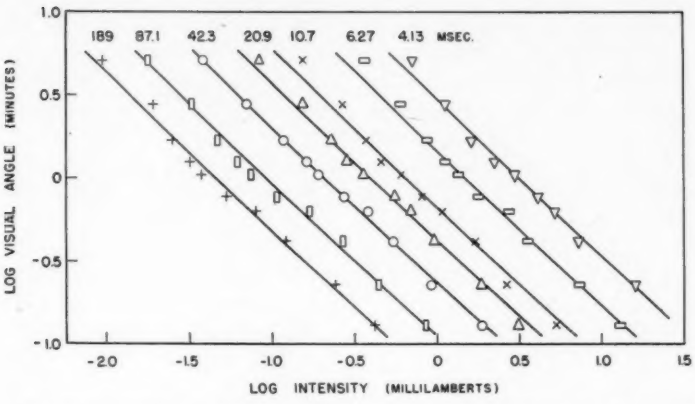

FIG. 1. The relation between logarithm of the visual angle in minutes subtended by the illuminated stimulus slit and logarithm of threshold intensity in millilamberts.

RESULTS

Table I presents the logarithm of the threshold intensity for resolution of the 10 illuminated stimulus slits. Each entry is the average of eight threshold determinations and is tabulated under the appropriate stimulus duration which varies from 189 to 4.13 msec. The width of the stimulus slit is indicated by the logarithm of the visual angle it subtends at the eye. In general, it may be noted, the mean threshold intensities display a satisfactory correspondence for the two subjects, although the values for JIN are consistently lower than those for RHB. Since there is consistent correspondence in the data, they have been averaged and the mean of each pair is presented in our figures for convenience of examination.

Figure 1 presents these values plotted as logarithm of the visual angle in minutes against logarithm of the intensity in millilamberts. The curve fitted to the experimental points was derived for the average values of our data, irrespective of duration, from the theoretical formulation presented by Hecht and Mintz.[3] It was then translated to the right and left on the intensity axis to give the best fit at each duration. The double log plot has the several advantages of showing deviations of equal percentage throughout the entire range, making the shape of the curve independent of the particular units involved, and bringing more clearly into view any discontinuity in the data caused by rod and cone function.[11] For any given stimulus duration the visual angle required for resolution of the slit is greatest at the lowest stimulus intensities. With increases in intensity, the visual angle decreases, nearly as a linear function in Fig. 1.

The influence of stimulus duration on the relation between visual resolution and intensity is clearly evident in Fig. 1. The exposure time has no effect on the shape of the curve itself, since at all durations the experimental points are fitted equally well by the same curve. There is, however, a definite and progressive shift to the right along the abscissa in the direction of increased threshold intensities of resolution with each decrease in duration. At a given intensity of illumination, the width of the illuminated slit which can be just resolved decreases as the interval of time during which the slit is exposed to the eye increases.

Since the observations were made with the foveal regions activated by the stimulus flash appearing in the black center of the stimulus configuration with its illuminated surround, there is no rod portion of the curves in Fig. 1. Although the greater irregularity of experimental points with the longer durations and lower intensities (0.3−0.01 ml) may be interpreted in terms of uncontrolled peripheral vision, our method and the absence of a sharp inflection point argue against such an interpretation. By and large, the data are continuous and do not exhibit the break

[11] S. Shlaer, J. Gen. Physiol. **21**, 165 (1937).

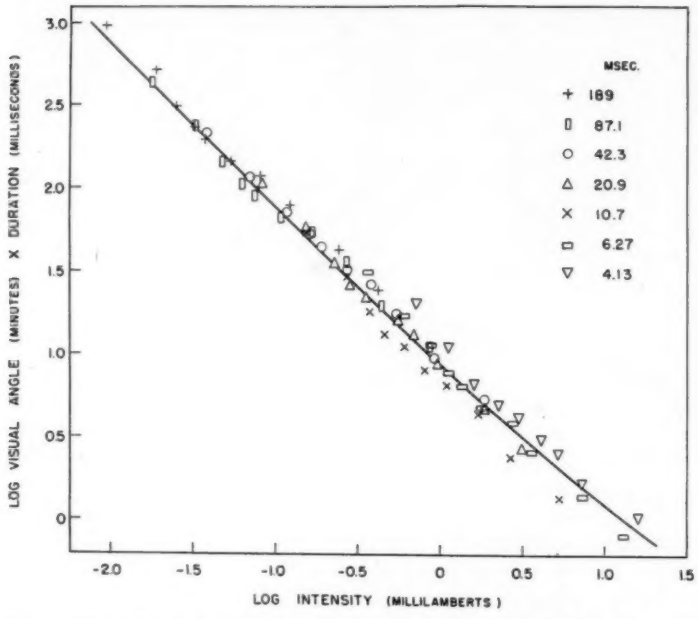

FIG. 2. The relation between $\log \alpha \cdot \tau$ and $\log I$. The curve is theoretical and derived from Hecht's theory of visual intensity discrimination.

found when measurements are made for the whole eye.

The results presented in Fig. 1 may be readily interpreted in terms of Hecht's theory of intensity discrimination.[4] Thus, Graham and Kemp[5] have found that intensity discrimination for the human fovea may be described by the equation

$$(\Delta I/I)\cdot\tau = C[1+1/(KI)^{\frac{1}{2}}]^2, \qquad (1)$$

where ΔI is the just discriminable increment of intensity, I the prevailing intensity, τ the duration of the stimulus flash, and C and K are constants. For the present we may disregard the situation for stimulus flashes of longer duration than the critical duration (τ_c), where the same equation applies but with τ_c substituted for τ.

Hecht and Mintz[3] have found that the visual resolution of an opaque line varies with intensity of the illuminated background as does the capacity for intensity discrimination, or as a first approximation

$$\alpha = b'\cdot\Delta I/I, \qquad (2)$$

where α is the visual angle subtended at the eye by the width of the stimulus and b' is a constant of proportionality.

Equation (1) may now be rewritten to apply to our data in the form

$$\alpha\cdot\tau = b[1+1/(KI)^{\frac{1}{2}}]^2, \qquad (3)$$

where $b=b'C$ and τ is a constant contained in Hecht's b for constant duration less than the critical duration. In our experiment the duration τ varies and, with the constants b and K, determines the position of the curve of Fig. 1. Thus, b/τ fixes the curve on the ordinates, K on the abscissas. When $\log \alpha$ is plotted against $\log I$, the shape of the curve is independent of particular values for τ, b, and K.

The extent to which the above analysis applies to our data may be tested by plotting $\log \alpha\cdot\tau$ against $\log I$. When we plot values of $\log \alpha\cdot\tau$ on a graph with $\log I$ as abscissa, we may expect to find a family of superimposed curves.

Figure 2 presents our data treated in this way. All values for the visual angle subtended by the width of illuminated slit have been multiplied by the appropriate stimulus duration. It may be observed that a single theoretical equation[12] de-

[12] We have derived by the method of averages from our data and Eq. (3) a value of 0.0670 for b and of 0.00906 for K. The theoretical curve drawn through the points of Fig. 2 is the equation $\alpha\cdot\tau = 0.0670[1+1/(0.00906I)^{\frac{1}{2}}]^2$.

scribes our family of superimposed experimental points with reasonable precision. The data for the seven curves of Fig. 1 fit the theoretical curve of Fig. 2 more than adequately in terms of experimental error and the small range of intensities with which we have worked.

On the basis of Hecht's suggestion that visual resolution is a form of intensity discrimination, we might expect to find not only the satisfying fit of our data by his equation, but also the occurrence of a critical duration beyond which further increases in stimulus duration would have no effect on the threshold intensity for resolution of an illuminated slit. Thus, Graham and Kemp[5] and Keller[6] have demonstrated the influence of a critical duration on intensity discrimination. When $\log \Delta I \cdot \tau$ is plotted against $\log I$ for various durations, the curves are displaced on the ordinate in a regular and parallel progression as long as the stimulus duration remains less than the critical duration. When the exposure time becomes equal to or greater than the critical duration, further displacement ceases and the succeeding curves are superimposable, but no longer parallel with the curves for durations less than critical. Our expectation of a similar effect in the present study of visual resolution is strengthened by its occurrence when the threshold light intensity is measured for acuity gratings at longer exposure times.[7]

Up to the longest duration of 189 msec. there is no indication that the critical duration has been reached in this experiment. The displacement of the curves of Fig. 1 continues unchanged for durations varying from 4.13 to 189 msec. The apparent absence of a critical duration may be interpreted in terms of the narrow width of our illuminated stimulus slits. As stimulus area decreases, the critical duration increases.[13] With small areas the critical duration is determined by the activity of relatively few elements and is correspondingly long. If the receptive units stimulated have comparatively long critical durations, the phenomenon may not be evoked within the limited range of durations we have used. Undoubtedly, if longer durations had been available for the present experiment, the effects of critical duration would have appeared.

SUMMARY AND CONCLUSIONS

This paper describes a method for studying visual resolution of an illuminated slit against a dark ground. A surrounding and uniformly illuminated circular area provides for consistent foveal fixation and stable retinal adaptation. As the width of the stimulus slit decreases, the threshold light intensity required for its resolution increases. For a given stimulus slit, the threshold intensity for resolution also increases as the stimulus duration decreases. These results are interpreted quantitatively in terms of Hecht's theory of visual intensity discrimination by considering stimulus duration a parameter of Hecht's equation. Thus, the present experiment extends the interpretation of visual resolution in terms of retinal brightness differences to the special case of an illuminated bar seen as test object against a black ground and to visual resolution-intensity functions for varying exposure times of stimulus.

[13] C. H. Graham and R. Margaria, Am. J. Physiol. **113**, 299 (1935); and H. W. Karn, J. Gen. Psychol. **14**, 360 (1936).

The Transient Stiles-Crawford Effect

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(Received August 23, 1944)

I. INTRODUCTION

THE Stiles-Crawford effect for steady-state foveal vision was formulated mathematically in a previous paper.[1] All research on the effect has been confined to equilibrium conditions, but situations sometimes arise in which it is desirable to consider the transient state. For example, in the study of dark adaptation, where curves are obtained for helios threshold *vs.* time, one finds that some data have been obtained with natural pupil and some with artificial pupil. Such researches cannot be compared with any certainty unless data are available on pupil diameter as a function of time during the opening of the pupil. Fortunately, experimental data are available on opening and closing of the pupil. Reeves[2] obtained this information in 1918, and some corroborative work on the subject was done recently by Crawford.[3]

TABLE I. Pupil diameter (closing), Reeves (Av. of 6).

t (sec.)	δ (mm)	$(\delta-\delta_{min})/(\delta_{max}-\delta_{min})$ Obs.	Calc.
0.0	8.0	1.00	1.00
0.1	7.8	0.96	0.99
0.2	7.6	0.92	0.90
0.3	7.2	0.84	0.82
0.4	6.8	0.76	0.74
0.5	6.4	0.67	0.67
1.0	5.3	0.45	0.41
1.5	4.4	0.27	0.25
2.0	3.9	0.16	0.15
2.5	3.5	0.082	0.090
3.0	3.3	0.041	0.055
3.5	3.2	0.020	0.033
4.0	3.2	0.020	0.020
4.5	3.1	0.0	0.012

We have obtained mathematical expressions for pupil size as a function of time, basing our equations on Reeves' data. For rod vision, the effective luminous flux to the retina is directly proportional to pupil area and is thus calculated very easily from the new expression for pupil diameter as a function of time.

For cone vision in the steady state, *the effective flux to the retina does not vary directly as the pupil area* but in a much less marked manner. This is the Stiles-Crawford effect.[4] It seems reasonable to extend the steady-state effect and to apply the equations also to the transient case. Equations and curves are given, by means of which the investigator can transpose his results from natural to artificial pupil and conversely. An application to dark adaptation shows that the transient effect of the pupil has a helpful influence in lowering the visual threshold immediately after the light is cut off and before photochemical adaptation in the retina has become effective.

2. OPENING AND CLOSING OF THE PUPIL

Pupil diameter in the transient state seems to have been measured by only two investigators: Reeves[2] in 1918 and Crawford[3] in 1936. Reeves employed a moving-picture camera, arranged to take a full-size picture of one eye. The pupil diameters δ were measured from the film and are the apparent sizes, as viewed from outside the eye, through the cornea and aqueous humor. In determining the pupil diameter δ as a function of time t when the pupil was closing, the subject was first adapted to complete darkness. The camera was then started, a large field having a helios of 1000 blondels (100 millilamberts brightness) was exposed, and a record was obtained of the change in pupil size as the pupil contracted from its maximum diameter to a new equilibrium state.

The data are shown in Fig. 1. The black disks represent averages for the six subjects. The white disks are individual readings and indicate the

[1] P. Moon and D. E. Spencer, J. Opt. Soc. Am. **34**, 319 (1944).

[2] P. Reeves, J. Opt. Soc. Am. **4**, 35 (1920); Psychol. Rev. **25**, 330 (1918).

[3] B. H. Crawford, Proc. Roy. Soc. London **B121**, 376 (1936).

[4] W. S. Stiles and B. H. Crawford, Proc. Roy. Soc. **B112**, 428 (1933).

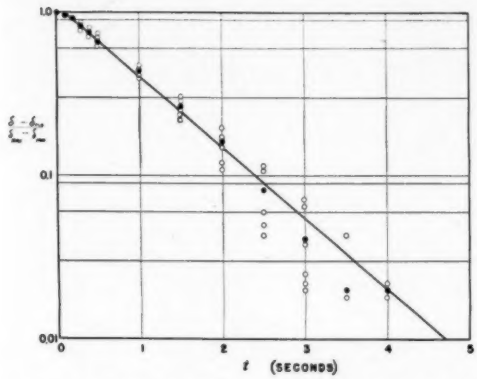

FIG. 1. Closing of the pupil. The vertical scale is logarithmic and thus the straight line represents an exponential function of time, Eq. (1). The black circles represent the Reeves data (Av. of 6 subjects) while the open circles are for individual subjects.

amount of scatter. At the smaller values of time, the individual points cluster so closely together on the diagram that most of them have been omitted. The average data are given also in Table I.

Evidently the Reeves results can be represented to within the experimental variation, by the straight line of Fig. 1. The equation of this line is

$$\frac{\delta-\delta_{\min}}{\delta_{\max}-\delta_{\min}}=1.10\exp(-t), \qquad (1)$$

where t = time in seconds after helios is increased to H_A; δ = pupil diameter (mm) at time t; $\delta_{\min}$ = minimum pupil diam., corresponding to equilibrium at helios H_A; $\delta_{\max}$ = maximum pupil diam., corresponding to equilibrium at $t<0$. The peculiar quantity on the left side of Eq. (1) was employed instead of δ because of its greater

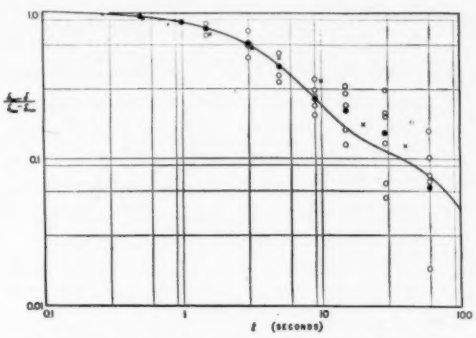

FIG. 2. Opening of pupil. ● Reeves (Av. of 6). ○ Reeves (individual subjects). × Crawford (Av. of 12). — Eq. (2).

generality. This characteristic will be noted later. Numerical values, computed and experimental, are compared in Table I. Equation (1) applies only for $t\geqq 0.10$ sec. For $t<0.10$ sec., the quantity $(\delta-\delta_{\min})/(\delta_{\max}-\delta_{\min})$ is taken as unity. The lag in the closure of the pupil is of course caused by the finite time required for the nervous impulses that finally initiate the muscular movement of the iris.

Similar results were obtained for opening of the pupil. Initial adaptation was to a constant helios $H_A=1000$. The same six subjects were used. Results are given in Fig. 2 and Table II. The first few data beyond $t=0.2$ sec. are represented by an exponential function as before, but further data do not fit this simple equation. The sum of two exponentials, however, gives a satisfactory fit, as shown in Fig. 2. The curve represents the equation,

$$\frac{\delta_{\max}-\delta}{\delta_{\max}-\delta_{\min}}=0.880\exp(-0.228t)+0.160\exp(-0.0125t). \qquad (2)$$

TABLE II. Pupil diameter (opening).

Reeves (Av. of 6) $\delta_{\max}=7.6$ mm, $(\delta_{\max}-\delta_{\min})=4.7$ mm				Crawford (Av. of 12) $\delta_{\max}=7.0$ mm, $(\delta_{\max}-\delta_{\min})=4.1$ mm			
		$(\delta_{\max}-\delta)/(\delta_{\max}-\delta_{\min})$				$(\delta_{\max}-\delta)/(\delta_{\max}-\delta_{\min})$	
t (sec.)	δ (mm)	Obs.	Calc.	t (sec.)	δ (mm)	Obs.	Calc.
0.0	2.9	1.00	1.00	0.0	2.9	1.00	1.00
0.5	3.1	0.96	0.94	0.27	3.0	0.98	0.99
1.0	3.5	0.87	0.86	0.53	3.3	0.90	0.94
1.5	3.9	0.79	0.78	0.80	3.6	0.83	0.89
3	4.7	0.62	0.60	1.6	4.1	0.71	0.77
5	5.6	0.43	0.43	3.2	4.7	0.56	0.58
9	6.4	0.26	0.26	5	5.0	0.49	0.43
15	6.6	0.21	0.16	10	5.6	0.34	0.23
29	6.9	0.15	0.11	20	6.3	0.17	0.13
60	7.3	0.064	0.075	40	6.5	0.12	0.097
180	7.6	0.0	0.017	80	7.0	0.0	0.060

Equation (2) applies to $t \geqq 0.20$ sec. For $t < 0.20$ sec., take $(\delta_{max} - \delta)/(\delta_{max} - \delta_{min}) = 1.00$. A very good fit is obtained for the first six points (see Table II). Further points show some discrepancy between calculated and experimental values, but these discrepancies are still well below the scatter of the individual points and are largely the result of using the quantity $(\delta_{max} - \delta)/(\delta_{max} - \delta_{min})$ which magnifies errors when it becomes small.

Crawford[3] employed an opal lamp in the field of view, in place of the large uniform field used by Reeves. There were twelve subjects. Measurements were made on opening of the pupil only, and all the data were obtained in complete darkness by means of infra-red photography. The average data are given in Table II and are indicated by the crosses in Fig. 2. A reasonably close agreement is found between the two sets of data. We have designed the empirical equations to fit the Reeves data, since these are the only data that apply to both opening and closing of the pupil.

Reeves' data, however, are all obtained with one value of helios, $H_A = 1000$. Suppose that a different helios had been used, so that the range of pupil diameters is completely different. How

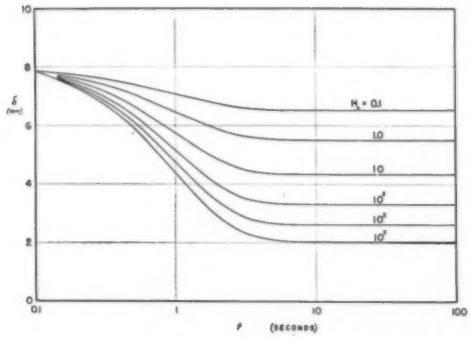

FIG. 3. Calculated curves of pupil diameter δ (mm) as a function of time (sec.). The eye was initially dark-adapted with the pupil at its maximum diameter. At $t = 0$, a uniform field of helios H_A (blondel) was suddenly exposed. The pupil remains at its original diameter until $t = 0.10$ sec., at which time it begins to decrease in size as shown by the curves.

are the various curves related? The simplest assumption would be that the constants of Eq. (2) are unaltered when H_A is changed. This assumption would correspond to a set of δ *vs.* t curves that could be superposed by changes in the vertical scale and by vertical translations. But such a set of curves was obtained experimentally by Crawford.[5] Three values of H_A are used, corresponding to initial pupil diameters of 3.3, 5.5, and 6.2 mm. The time in the dark,

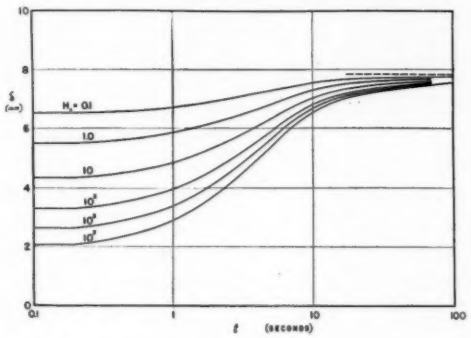

FIG. 4. Calculated curves of pupil diameter as a function of time. The eye was initially adapted to a large surface of helios H_A. At $t = 0$, this helios was suddenly reduced to zero. Pupil size remains constant for 0.20 sec., after which it changes according to the curves. The final pupil diameter reached in all cases is indicated by the dotted line.

required to reach an approximately final value of 8.0 mm, is the same in all three cases. We assume, therefore, that *Eq. (2) is valid for any range of pupil diameter.* The same assumption will be applied also to Eq. (1), though there is as yet no direct experimental evidence for the assumption in this case.

Equations (1) and (2) are therefore regarded as general expressions, applying to all cases of pupil variation. They can be employed in calculating average curves of pupil diameter as a function of time for any initial and final values of helios. Figure 3, for example, exhibits a family of curves for the condition that (at $t = 0$) the helios is suddenly changed from zero to H_A. A similar set for the opening of the pupil is given in Fig. 4. The outstanding feature of these curves, as pointed out by Reeves for his curves, is the much greater speed of closing than of opening. For the closing of the pupil, *the time constant is 1.0 sec.*, as can be seen from Eq. (1). For opening, the same ordinate is obtained when

$$1.040/2.718 = 0.880\ (\exp(-0.228t) + 0.160 \exp(-0.125t),$$

[5] Reference 3, Fig. 9, p. 393.

which is satisfied by $t=5.7$ *sec*. Thus, closure of the pupil is approximately six times as rapid as opening. Since Eq. (2) is not a single exponential function, there is no true time constant in opening and the foregoing ratio depends on t, being 4.4 at $t=0$ and 31.6 at $t=100$ sec. In any case, the statement that is often made[2]—that the ratio is 60—is misleading.

It is interesting to note that both the action of the pupil and the action of the retina itself are such as to protect the retina from a sudden increase in helios. The pupil closes much more rapidly than it opens. Simultaneously, the retina adapts itself to an increase in helios, which it does much more rapidly than it adapts itself to a decrease in helios.[6]

3. THE STILES-CRAWFORD EFFECT

One reason for studying the pupil diameter as a function of time is to find how the effective amount of light reaching the retina varies when the pupil opens or closes. For rod vision, the effective amount of light varies directly as the pupil area and is easily calculated as a function of time by use of the equations or curves of Section 2. For cone vision, however, the Stiles-Crawford effect[4] makes the calculation somewhat more complicated.

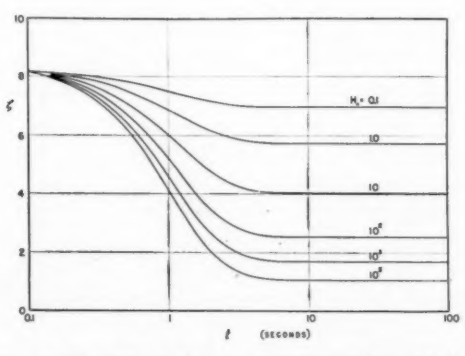

FIG. 5. The Stiles factor as a function of time (closing pupil). Pupil initially at maximum size. Cone vision.

In a previous paper[1] the *Stiles factor* $\zeta(\delta, \beta)$ was defined as the ratio of the effective luminous flux with pupil diameter δ to the effective luminous flux with pupil diameter β, or

$$\zeta(\delta, \beta)=\frac{\int_0^{\delta/2}\eta(x)xdx}{\int_0^{\beta/2}\eta(x)xdx}. \quad (3)$$

The quantity $\eta(x)$ is a weighting factor which expresses the relative effectiveness of luminous flux entering the pupil at x mm from its center:

$$\eta(x)=F(0)/F(x).$$

For the rods,

$$\zeta(\delta, \beta)=(\delta/\beta)^2. \quad (4)$$

When the pupil changes from its smallest to its largest diameter, the Stiles factor for rods changes by a factor of approximately 15.

For cone vision, the Stiles factor never exceeds approximately 8. Rays that enter near the edge of the pupil are much less effective photometrically than those entering near the center. There is considerable evidence that the phenomenon occurs in the retina and is associated with the difference in angle at which the rays pass through the retina layer. As shown previously,[1] the experimental data can be represented by the equation.

$$\zeta(\delta, \beta)=\left(\frac{\delta}{\beta}\right)^2\frac{[1-A\delta^2/8+B\delta^4/48]}{[1-A\beta^2/8+B\beta^4/48]}, \quad (5)$$

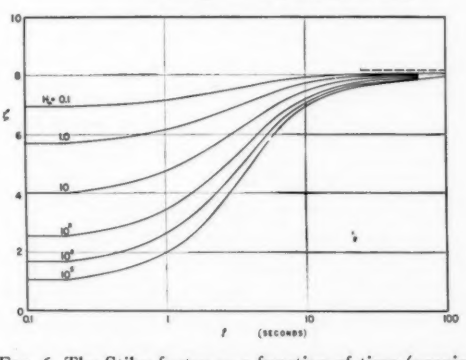

FIG. 6. The Stiles factor as a function of time (opening pupil). Eye initially adapted to helios H_A. Cone vision.

If $\beta=2.0$ mm, the Stiles factor becomes

$$\zeta(\delta, 2)=0.2620\delta^2[1-1.06\times10^{-2}\delta^2+4.17\times10^{-5}\delta^4]. \quad (6)$$

Experimental data for the Stiles-Crawford effect have been obtained under equilibrium con-

[6] P. Moon and D. E. Spencer, "Visual dark adaptation: a mathematical formulation," to be published.

ditions. In fact, no one seems to have even thought of a transient Stiles-Crawford effect. However, if the effect is caused merely by greater absorption or reflection of rays that pass through the retina obliquely, then no time factor is involved and the data and equations obtained in the steady state will apply also in the transient state. The transient Stiles-Crawford effect will be calculated, therefore, by substituting Eq. (1) or Eq. (2), which expresses the pupil diameter as a function of time, in Eq. (6) for the Stiles factor. Calculated results are shown in Figs. 5 and 6. These curves apply to cone vision only. With rod vision there is no Stiles-Crawford effect, and the Stiles factor is obtained from Figs. 3 and 4 and Eq. (4).

With either rods or cones, the procedure in the use of the Stiles factor is very simple. If data are known for natural pupil, they can be transposed to the condition of an artificial pupil of diameter β by a suitable change in the helios:

$$H_\beta(t)=\zeta(\delta,\beta;t)H_\delta(t), \tag{7}$$

where H_β = helios for artificial pupil of diameter β; H_δ = helios for natural pupil of diameter δ. Obviously Eq. (7) can also be used in changing from data taken with an artificial pupil to the corresponding results for natural pupil.

4. DARK ADAPTATION

One of the principal applications of the foregoing analysis is to dark-adaptation curves. Such curves are sometimes obtained with a natural pupil and sometimes with an artificial pupil, and it is necessary to be able to change from one form to the other. The curves and graphs of the previous sections can be employed.

In a study of dark adaptation,[6] we obtained equations representing the dark-adaptation threshold as a function of time for all degrees of previous light adaptation. The simplest case occurs when the eye has been light-adapted to a constant helios H_A for sufficient time so that it has reached an equilibrium condition. For the rods, the concentration of retinene is given by the equation

$$x=x_0[\exp(3k_2t)-k_4x_0^3[\exp(3k_2t)-1]/k_2]^{-\frac{1}{3}}, \tag{8}$$

where x = instantaneous concentration of retinene in a rod, expressed as a fraction of maximum possible concentration; x_0 = concentration at $t=0$; t = time (sec.); $k_2=2.00\times10^{-3}$; $k_4=1.98\times10^{-3}$. The quantity x_0 is calculated from the equation,

$$H_A=\frac{(k_2-k_4x_0^3)ax_0}{ak_1[\exp(-ax_0)-\exp(-a)]}, \tag{9}$$

where $a=9.0$.

After x is calculated, the threshold helios, which is just detectable at time t, is found from the relation

$$(H_T/H_{T\infty})_2=[\exp(-ax)-\exp(-a)]^{-1}. \tag{10}$$

The quantity $H_{T\infty}$ is the threshold helios when $t\to\infty$. It is often called the *absolute threshold* and

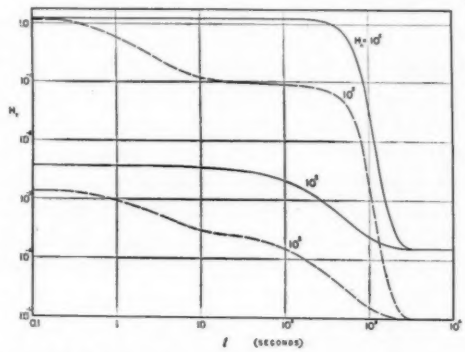

FIG. 7. Calculated dark-adaptation curves showing the effect of an artificial pupil. Threshold helios (blondels) is plotted against time (sec.). Rod vision. — 2-mm artificial pupil. -- natural pupil.

will be assumed to have the value 15.3×10^{-5} blondel for a 2-mm pupil. The subscript 2 in the expression $(H_T/H_{T\infty})_2$ refers to the fact that the data from which the numerical constants were determined were obtained with an artificial pupil 2 mm in diameter. All the equations and curves of the paper on dark adaptation[6] are obtained with the 2-mm pupil. Two dark-adaptation curves calculated in this manner for rod vision are shown in Fig. 7.

We now apply the methods of the preceding sections to predict corresponding dark-adaptation curves with the natural pupil. The helios ratio with natural pupil will be called $(H_T/H_{T\infty})_n$. The $H_{T\infty}$ in this case is the absolute threshold for the natural pupil and has a value of approxi-

mately 1.0×10^{-5} blondel for rod vision. At any value of time, the threshold for natural pupil is related to the threshold for artificial pupil by the equation

$$H_{Tn}=H_{T2}/\zeta. \tag{7a}$$

For the particular case of the absolute threshold, $(H_{T\infty})_n=(H_{T\infty})_2/15.3$, since $\zeta=15.3$ for the average equilibrium pupil size in darkness. Thus

$$(H_T/H_{T\infty})_n=\left(\frac{H_T}{H_{T\infty}}\right)_2\left(\frac{15.3}{\zeta}\right). \tag{11}$$

Further equations used in the computation of Fig. 7 are

$$\begin{aligned}\log (H_T)_2&=\log (H_T/H_{T\infty})_2+6.184-10,\\ \log (H_T)_n&=\log (H_T/H_{T\infty})_2+\log (15.3/\zeta)\\ &\quad +5.00-10.\end{aligned} \tag{12}$$

The computed curves for rod vision with 2-mm pupil and with natural pupil are compared in Fig. 7 for two values of adapting helios. The retina is thoroughly light-adapted to a helios of 10^5, and at $t=0$ it is put in complete darkness. With the artificial pupil, the top curve shows that the threshold helios remains practically constant to 200 seconds. After this time, the curve falls sharply and reaches a final value at roughly 3000 seconds. With the natural pupil, the threshold is lower at all values of time. At $t=0.1$ sec. the two curves are close together because the natural pupil is only slightly larger than 2 mm. At large values of time, however, the artificial pupil is still at 2 mm while the natural pupil has reached almost 8 mm. In the latter case, the greater aperture allows a lower threshold. Another pair of curves is shown in Fig. 7 to indicate what happens at a lower adaptation helios.

Perhaps the most interesting feature of these curves is the quick action of the pupil in reducing the threshold before the photochemical process in the retina has become effective. According to our mathematical formulation,[6] the photochemical adaptation in the rods does not produce a marked effect before 100 sec. The pupil, however, changes quickly and in 10 sec. reduces the threshold to approximately $\frac{1}{10}$ of the value it would have with a 2-mm pupil. This ratio of $\frac{1}{10}$ is seen to remain practically constant for all values of t greater than 10 sec.

The statement is often made that the 16:1 ratio of retinal illumination produced by maximum pupil change can have very little effect in visual adaptation, where the helios range may be 10^6:1, and that therefore the change of pupil size is merely a complication which serves no useful purpose. Figure 7 illustrates, however, what is probably an important function of the pupil; namely, the quick reduction in the threshold.

The need for such a pupil change is completely masked by the usual method of plotting dark-adaptation curves on a linear time scale instead of a logarithmic scale. Hecht's curves,[7] for instance, seem to have almost an infinite slope as t approaches zero. When plotted as in Fig. 7, however, the curves show that the photochemical process is of no help for several seconds, during which time the pupil is effective in reducing the threshold. Evidently this rapid pupil change is of value.

Another conclusion to be drawn from Fig. 7 is that, for rod vision and values of time greater than 10 sec., the quantity $(H_T/H_{T\infty})$ will be practically the same whether obtained with artificial or natural pupil. Since most rod adaptation data apply only to greater values of time than 10 sec., the curves of $(H_T/H_{T\infty})$ may be obtained in either manner. *Most dark-adaptation research on rod vision can be made equally well with natural pupil or artificial pupil.* Curves of $(H_T/H_{T\infty})$ will be practically identical in the two cases, and curves of H_T will differ by a constant factor. This conclusion, however, does not apply to rod vision with $t<10$ sec. or to most cases of cone vision.

Curves for cone vision are calculated in a similar manner, though the equations are somewhat simpler because of the lower concentration of photochemical decomposition products in the cones. The concentration of photochemical decomposition product for a cone is

$$x=x_0\exp(k_2 t). \tag{13}$$

The equilibrium value of x is found from the equation,

$$H_A=(k_2/ak_1)(ax_0)\exp(ax_0), \tag{14}$$

and

$$(H_T/H_{T\infty})_2=\exp(ax). \tag{15}$$

[7] Hecht, Haig, and Chase, J. Gen. Physiol. **20**, 831 (1927).

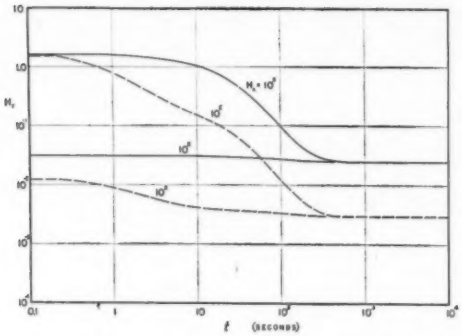

FIG. 8. Calculated dark-adaptation curves. Cone vision. — 2-mm artificial pupil. -- natural pupil.

In the equations for cone vision,

$$ak_1=3\times10^{-5}, \quad k_2=0.0110, \quad a=30.$$

The change from artificial to natural pupil is given by the relation,

$$(H_T/H_{T\infty})_n=(H_T/H_{T\infty})_2(8.17/\zeta). \qquad (16)$$

The thresholds are $(H_{T\infty})_2=24.5\times10^{-3}$ and $(H_{T\infty})_n=3.0\times10^{-3}$, so

$$\log (H_T)_2=\log (H_T/H_{T\infty})_2+8.389-10,$$

$$\log (H_T)_n=\log (H_T/H_{T\infty})_2+\log (8.17/\zeta) +7.477-10. \qquad (17)$$

Typical curves for cone vision are shown in Fig. 8. When the eye is adapted to 10^5 blondels and a 2-mm artificial pupil is used, the threshold helios is seen to be almost constant for 1 sec. It then falls rapidly and reaches a final value (the absolute cone threshold) in approximately 500 sec. The range of H_T for the cones is seen to be considerably less than for the rods. For $H=10^5$, the range is of the order of 100:1 for cones and 10,000:1 for rods. This fact is in accordance with the experimental findings of Hecht[7] and other investigators. With a natural pupil, the dark-adaptation curve descends more rapidly and reaches a lower absolute threshold than with an artificial pupil. When the adapting helios is low ($H=100$ in Fig. 8), the cones exhibit hardly any effect of dark adaptation, and what little effect exists is caused by the pupil rather than by the photochemical process in the retina. As with the rod curves, the effect of the pupil is practically constant for $t>10$ sec., so curves of $(H_T/H_{T\infty})$ will be essentially the same whether taken with artificial or with natural pupils. With cone vision, however, dark adaptation is so rapid that the portion of the curve to the left of $t=10$ sec. may be of importance. Figure 8 shows that for $t\leqq10$ sec., the two dark-adaptation curves are distinctly different.

5. SUMMARY

The paper extends the previous investigation of the Stiles-Crawford effect to the transient state. Empirical equations are first developed to represent Reeves' data on the opening and closing of the pupil. These equations are then substituted into the previously determined equations for the Stiles factor which relates the helios threshold with natural pupil to that with artificial pupil. In this way, data obtained in the transient state with artificial pupil can be transformed to equivalent results for natural pupil.

The principal use of the foregoing method is in dark-adaptation curves. Examples of such curves are plotted for a 2-mm artificial pupil, and these curves are then modified so that they apply for natural pupil. The outstanding features are:

(1) The curves of relative threshold $(H_T/H_{T\infty})$ are practically identical in the two cases if values of time less than 10 sec. are not employed.

(2) The photochemical adaptation of the retina is comparatively slow in starting. During this period of comparative inaction of the photochemical process, there is a rapid change in pupil size. This appears to be the true function of the pupil: to reduce the threshold swiftly before photochemical adaptation has commenced. It enables a person to step from bright sunlight into a dark room and achieve some degree of adaptation within 5 seconds. True, the resulting state at $t=5$ sec. is far from complete dark adaptation; but it is much better than no change, and to reach it would take from 10 to 100 times as long if the pupil were fixed.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 34, NUMBER 12 DECEMBER, 1944

Direct Reading Instrument for Spectrochemical Analysis

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(Received September 18, 1944)

The conventional method of quantitative spectrochemical analysis employing bright-line spectra and photographic recording is discussed with reference to the various steps involved. The steps which may be eliminated by the application of direct photoelectric recording are considered and a method based on this type of recording is discussed. A detailed comparison of the two methods is made which leads to the conclusion that only by designing an entirely new spectrum-dispersing instrument can "direct reading" be made sufficiently flexible for general adaptation. Such an instrument is considered in detail together with suitable analytical procedure for its use. The probable speed and accuracy attainable are illustrated by examples. Present limitations and future developments are considered. A proposed instrument is announced.

ANYONE who has ever used spectrographic methods for chemical analysis extensively has undoubtedly wondered why an instrument could not be built which would give, directly on meters, accurate quantitative chemical analyses of all elements to be measured—in a fraction of a minute per sample. A number of workers have pointed out that such instruments can now be built.[1–4] In fact by this time many doubtlessly are in existence. However, whether any of these experimental units is the prototype of the successful instruments of the future remains to be seen. That there will be successful instruments quite universal in their scope on which most routine quantitative analyses will ultimately be conducted is a certainty. However, at this time there still exist numerous practical limitations which should be clearly understood so that unwarranted optimism concerning replacement of the more or less standard qualitative and quantitative systems of today, employing the photography of bright-line spectra and the densitometric measurement of such a record, will not be created.

Before discussing these limitations, however, it will be worth while to survey the great advantages of a direct reading system of analysis as compared to the photographic system. In order to appreciate clearly the difference, let us consider the two systems side by side in Table I.

A comparison of these figures shows that in general direct reading is about four times as rapid as the present photographic process even including the time of sample preparation. In obtaining these figures it is assumed that each system has been calibrated and that a single operator is doing the work. In actual practice, up to four operators, including the worker who prepares samples, may be used with the photographic system and two with the direct reading system. However, more operators merely keep all of the equipment operating more of the time and cannot materially cut down the time from start to finish of the analysis of a single sample, which is the time indicated above.

Thus two outstanding advantages beckon in the development of direct reading equipment. The first is connected with so called "speed analysis." This provides a "to-pour or not-to-pour analysis" in the case of metal production, in a minimum of time. Every minute saved is a minute less of furnace time required. Cutting such an analysis from ten minutes to two-and-a-half minutes from the time the sample is received, provides real economies in fuel, overhead costs, etc. The other advantage is related to technician costs and laboratory overhead per analysis. If these can be reduced from say eight

[1] G. R. Harrison, "Some future possibilities of spectrographic analysis," Conference on Spectroscopy and Its Applications, Cambridge (1940).
[2] O. S. Duffendack and W. E. Morris, J. Opt. Soc. Am. **32**, 8–24 (1942).
[3] D. H. Rank, R. J. Pfister, and P. D. Coleman, J. Opt. Soc. Am. **32**, 390–396 (1942).
[4] E. A. Boettner and G. P. Brewington, J. Opt. Soc. Am. **34**, 6–11 (1944).

TABLE I. Spectrochemical analysis.

Present photographic system Steps	Time	Direct reading system Steps	Time
Preparation of sample	1.5 min.	Preparation of sample	1.5 min.
Discharge to sample and photography	0.75 min.	Discharge to sample	0.75 min.
Develop, wash, and dry film or plate	5 min.	Read instrument for each element and record percentages	$0.05n$ min.
Place film in comparator-densitometer, find lines, read trans. values for internal standard line and each element line	$(0.7+0.2n)$ min.		
Convert trans. values to percentage composition	$0.2n$ min.		

(where n is the number of elements being determined)

	Total time in minutes									
Number of elements	1	2	3	4	5	6	7	8	9	10
Photographic analysis	8.3	8.7	9.1	9.5	9.9	10.3	10.7	11.1	11.5	11.9
Direct reading analysis	2.30	2.35	2.40	2.45	2.50	2.55	2.60	2.65	2.70	2.70

(These figures represent very approximate estimates.)

cents per element determined to two cents, obviously this is a worth-while economy.

However, before becoming too enthusiastic, it would be well to examine some of the limitations of the direct reading system. In the first place, careful analysis of the problem shows that it will not be expedient merely to attach a few gadgets on present spectrographic equipment and make it direct reading. This does not mean that it is impossible to do this but rather that the flexibility of such a system will be so limited that it will ultimately lose out in competition with apparatus built for the job. Thus, in general, direct reading will mean new equipment—at least to replace the spectrograph.

In order to see clearly the lack of flexibility that conversion of a standard spectrograph would give, consider the space requirements of direct reading equipment with respect to the spectrum, if the assumption is made that a photoelectric cell or multiplier will be used to receive the light. It is true that Geiger counters can be used particularly in the far ultraviolet but there is some doubt about whether satisfactory counters can be constructed for the visible and infra-red spectrum. In any event, unless one resorts to extremely small diameter counters—which pose many new problems—little space is gained over using photo-cells or multipliers.

Thus starting with photo-cell dimensions of 31 mm in diameter, it is apparent that at least 220A of the spectrum will be covered up if the photo-cell is put directly in the spectrum beam. This is on the basis of a spectrograph of constant dispersion, 7A per mm. Besides the photo-cell, room for a surrounding case must be provided so that the practical minimum length of the spectrum covered by a photo-cell is about 400A. It might appear that with a spectrograph covering 2220A at the camera, this would allow five photo-cell receivers, which would be sufficient for much work as an internal standard line and four element lines could be read. Unfortunately, however, the lines of the various elements will not in general be 400A apart and distributed uniformly throughout the spectrum.

Another approach to the problem is in the use of mirrors. Since the actual line to be measured is say 0.05 mm to 0.1 mm wide at the focal plane, the use of narrow mirrors in this plane should allow lines quite close together to be utilized. With mirrors, the light beam constituting a spectrum line can be reflected in four principal directions: up, down, to the left, and to the right. Of these, the first two are the most practical directions as groups of receivers can then be placed along the spectrum. How many receivers can be placed in a group? By moving the mirrors in front of the focal plane various distances, four

receivers can be accommodated in a very small region of the spectrum. This is clearly shown in Fig. 1, which is a view of four photoelectric receivers viewed from the grating end of an experimental direct reading instrument. Two receivers extend toward the camera from the mirrors, two away. The narrow mirrors are on the ends of the shafts fastened to the receivers.

One very important feature of this design is that the four receivers can pass by each other. The four mirrors used are 1.6, 2.4, 3.6, and 4.8 mm wide. Thus when these are all next to each other, spectrum lines 14A, 21A, and 30A apart

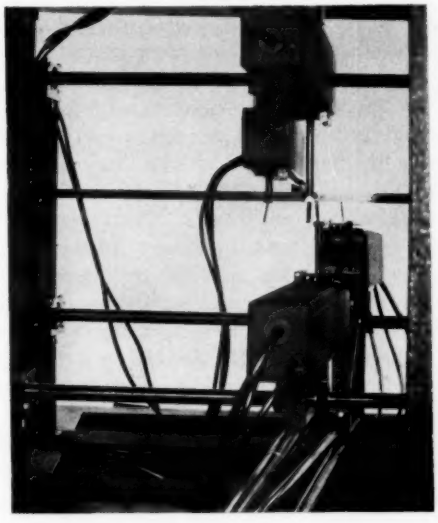

FIG. 1. Photoelectric receivers and mirrors in an experimental direct reading instrument.

can be used simultaneously. This is a considerable improvement over being limited to the use of lines 400A apart. Another very important point is that this design allows three groups of four receivers, or a total of twelve, to be used simultaneously. The only limitation on these other two groups is that the receivers of one group cannot pass by those of another group. Thus if we consider the receivers of the three groups next to each other, the width of each will determine the distance of closest approach. This is about 440A. Thus the final arrangement is quite flexible. The intensity of twelve lines may be measured simultaneously. Four of these lines may be distributed over 440A, with the second line being within 14A of the first, the third line within 21A of the second, and the fourth line within 30A of the third. In the next four, the same limitations hold with the additional one that the first receiver of this group must be 440A away from the first receiver of the previously-mentioned group. The same applies to the last group.

In this experimental model these receivers can be placed in the spectrum at any wave-length between 2000 and 6000A. This is accomplished by having large precision screws of 20 threads per inch carry each of the receivers. The screws are rotated by handwheels on the outside of the equipment. By counters and vernier dials on the screws, the positions of the receivers can be accurately followed.

From the foregoing, it should be quite obvious that, at least with the system discussed, a special instrument must be produced to replace the spectrograph. Conversion of a spectrograph to a direct reading instrument of this type would restrict its use so much that in general it may be considered impractical.

The next point of interest is, what system of analysis should be used? Should conventional source units be utilized to produce spectra? Should a receiver of the integrating type be used to replace the film? Should an internal standard line be employed? Should our calculating systems be patterned after the present one? These and many other questions come to mind. In general these questions should be answered in the affirmative wherever possible, as the closer accepted practice is followed, the more certain one can be of producing acceptable results.

Thus, the receivers developed are of the integrating type, giving integrated line intensities directly in terms of dial readings on counters. They employ photo-tube multipliers and suitable vacuum-tube amplifiers. In testing the unit, we have used a standard spark unit as the spectral source. In obtaining results, one receiver is placed in the path of an internal standard line and the other receivers in the paths of the lines of the elements to be determined. Determinations are made by having the counters calibrated in percentage composition. Adjustment for the internal standard line intensity is eliminated by sparking until a definite count or integrated intensity is

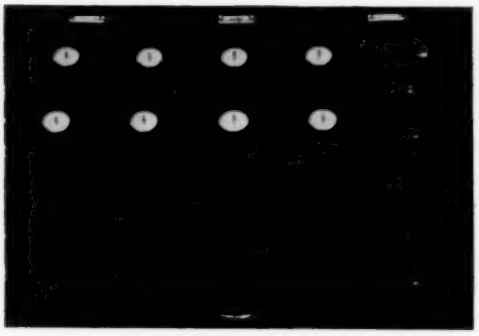

FIG. 2. Counter panel showing dials, each of which is calibrated directly in percentage composition of element being measured.

obtained for the internal standard line for each sample. Figure 2 illustrates the counter panel.

The limited results obtained to date on this unit have been very interesting. The first experiment was to determine how reproducible the measurements of an intensity ratio could be made. In order to eliminate source errors, a small lamp powered from a voltage regulator was used as the light source. Two receivers were placed near each other in the blue portion of the spectrum and counts taken for a 20-sec. period. Table II shows the counts obtained with the shutter on the spectrograph opened for 20 sec. with an electric timer. Background counts were obtained by operating the receivers for the same length of time with the shutter closed. This background is entirely electrical in nature and represents a new variable which must be considered. The analogous thing to it in photographic work is fog on the film. Whether it can be ignored just as fog is ignored, remains to be seen. A study of the results indicates that first the absolute reproducibility of the counts is quite good, varying only about 2 percent between extremes. The ratios between counts corrected for background show an average deviation from the mean of 0.57 percent. The ratios without background correction have an average deviation of 0.62 percent from the mean. Two conclusions seem in order from these results. The direct reading receivers used are capable of results approximately as accurate as those obtained photographically when several hundred counts can be used; electrical background correction probably need not be considered providing calibration can be made often enough and completely enough. However, it may be well to point out that ten photo-cell multipliers had to be tested in order to get two with this degree of stability.

The next step was to determine what slit widths are practical for a direct reading instrument. Once this is known, then the scope of this type of instrument can readily be predicted from photographed spectra and known analytical systems. Two slits need to be considered, the primary slit and the receiver slit. At first thought it may appear that the receiver slit should be identical in width to the spectrum line image which is determined by the primary slit width. This produces the highest resolving power for a given receiver slit width but it is entirely impractical for the simple reason that it is impossible to build an apparatus so fixed in its dimensions that a line once placed on the receiver slit will just fill that slit completely for even a reasonable length of time. A creep factor must be considered. There are two alternate schemes possible—either a wide primary slit and a narrow receiver slit should be used, or vice versa. In the first case a slight motion of the receiver slit merely moves it to another portion of the line image, while in the latter case the spectrum line image merely moves to another portion of the receiver slit. In either case little change of energy as received by the

TABLE II. Reproducibility of measurement.

Receiver I Counts			Receiver II Counts				
With light	Without light	Corrected	With light	Without light	Corrected	Ratio original counts	Ratio corrected counts
402	20	382	201	31	170	0.500	0.445
402	22	380	201	32	169	.500	.445
404	20	384	201	30	171	.498	.445
402	21	381	200	31	169	.497	.443
397	21	376	198	32	166	.499	.442
402	21	381	198	31	167	.493	.439
398	21	377	199	33	166	.500	.441
401	21	380	201	31	170	.501	.447
399	22	377	199	33	166	.499	.440
399	20	379	201	32	169	.504	.446
396	22	374	199	32	167	.504	.447
398	20	378	202	33	169	.507	.447
395	21	374	200	33	167	.506	.447
397	19	378	199	31	168	.501	.445
399	21	378	201	33	168	.504	.445
399	21	378	203	32	171	.509	.452
396	21	375	199	34	165	.504	.440
395	21	374	202	33	169	.512	.452
395	21	374	201	33	168	.509	.449
397	21	376	200	33	167	.504	.444

Average deviation from mean—original counts ±0.62 percent
Average deviation from mean—corrected counts ±0.57 percent

(Obtained with 6-volt lamp at primary slit operated from a voltage regulator)

photo-cell should be found as the receiver is moved slowly across the line. If these cases are analyzed with respect to ratio of line intensity to spectral background intensity, it will be found that the two cases are identical in this respect. What is gained by using a narrow primary slit is lost with a wide receiver slit and vice versa. As far as resolution is concerned, the same thing applies; exactly the same resolution is achieved by using a wide spectrum line and a narrow receiver slit as is obtained by interchanging the two in width. For ease of calculating, we have made the primary slit width—which is essentially the same width as the spectrum line it produces on our instrument—$\frac{1}{3}$ the receiver slit width. This allows a motion of the receiver with respect to the spectrum equal to the width of the spectrum line, before a change of reading is noted. With a spectrum line 0.1 mm wide and receiver slit 0.3 mm wide, two lines 2.1A apart can be theoretically resolved at all times, providing the apparent motion of the receiver with respect to the spectrum is less than 0.1 mm. In the present model we have successfully measured lines 2.2A away from the nearest adjacent line without interference and for a reasonable length of time. Figure 3 clearly shows that this can be done. The ordinates represent counts or intensities, the abscissa positions of the receiver with respect to the spectrum.

This brings up another important point. How rapidly can a receiver be placed accurately over a certain spectrum line? This will determine the flexibility of a direct reading instrument; whether it can be used to solve a wide variety of analytical problems, or whether it must be set up to do just one analytical job and left that way because of the tediousness of adjustment. Very limited experiments on this point seem to indicate that the screws controlling the positions of the receivers can bring the receivers within 0.5 mm of a spectrum line once the revolution counters have been calibrated in terms of wave-length. This corresponds to placing the receiver within 3.5A of the desired position. Once the spectrum is known, it should then be an easy matter, with the aid of standard samples, to decide when the counter is over the desired line. Then by taking a line-profile study, such as shown in Fig. 3, the exact operating position can be easily determined. All in all, this will take about five minutes per receiver, so that in a half-hour it should be possible to readjust the instrument to analyze for five elements in an entirely different alloy than was previously being analyzed. Besides adjusting the receivers, the system must be standardized. This will require about 2.5 min. per standard or 7.5 min. for three standards with five elements to be determined in each. Thus, in about three-fourths of an hour it should be possible to readjust the receivers and standardize a system of analysis for five elements in a different alloy than was being tested previously. This, of course, presupposes that an original setting-up period has been completed for each of the alloys so that the spectrum lines to be used and the number of counts to be expected under various conditions have been determined. However, even in the case of work being done for the first time, it should be possible to adjust the instrument from known spectrographic data in a matter of hours.

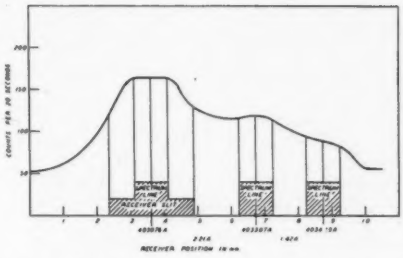

FIG. 3. Resolving power test. Profile study of manganese triplet showing complete resolution of two lines 2.21A apart and incomplete resolution of two lines 1.42A apart.

One last point should be discussed before actual analytical results are shown. What are the limitations of a direct reading instrument compared to a spectrograph with respect to minimum concentrations to be detected and measured? It is a well-known fact that the narrower the primary slit of a spectrograph is made, the greater is the ratio of line intensity to background intensity immediately adjacent and at the line. This is owing to the fact that much background is a continuous spectrum which increases in intensity approximately proportionately to the slit width while the spectrum line is merely widened proportionately to slit width, its intensity per unit area remaining about the same particularly after a certain slit width is reached. Because of these facts, it is common practice to narrow the spectrograph

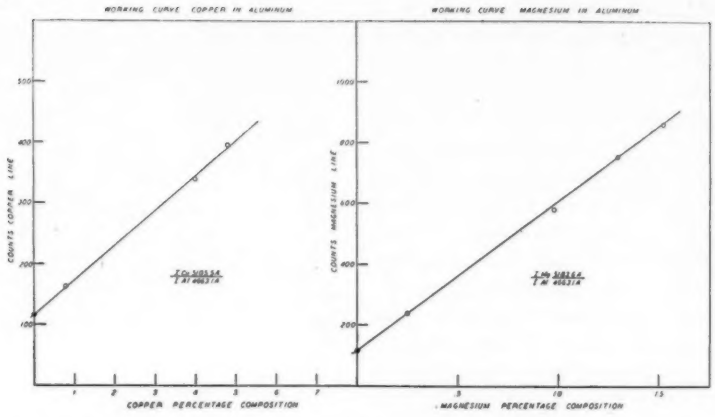

FIG. 4. Working curves for the quantitative determination of copper and magnesium in aluminum, obtained with the direct reading instrument.

slit in order to detect elements at very low concentrations.

Our laboratories have made it a practice for some time to define the limit of measurement as the percentage at which the intensity of the line of the element being determined, after background is subtracted, is 0.3 of background intensity as this still allows reasonably accurate determinations to be made. Since this varies with the primary slit width and the particular kind of spectrograph being employed, both of these facts should also be given.

How is all this related to direct reading? Obviously there is a minimum primary slit width required to provide enough light for the determination. Besides this, a considerably wider receiver slit is required to allow for receiver motion. The values used in the experimental model are 0.1 mm and 0.3 mm, respectively. This corresponds to a 10-division, 100-micron primary slit on an A.R.L.-Dietert spectrograph. However, three times as much background will enter the receiver slit as would be registered on the film as a part of the line blackening. Thus in general, three times the minimum concentration measurable by the above spectrograph with a 100-micron slit is required to be measured on the direct reading instrument. With these considerations in mind, some practical results can now be discussed.

The analysis of aluminum for magnesium and copper was tried first with the direct reading instrument. The aluminum line at 4663.1A, the copper at 5105.5A, and the magnesium at 5183.6A gave very acceptable results. Figure 4 shows the working curves for those cases. The ordinates are counts obtained from the copper and magnesium lines when a count of 400 was obtained from the aluminum line. This required a ten-sec. prespark and about a thirty-sec. spark using a standard spark unit with capacitance at 0.021 microfarad and inductance at 365 microhenries. The samples used were Alcoa standards sparked on a Petrey stand. The precision was such that 4 percent copper determinations could be made with an average reproducibility of ± 0.1 percent while 1 percent magnesium determinations differed on the average by ± 0.03 percent. These results appear very encouraging and prove conclusively the practicability of this direct reading instrument.

In order to determine just how far an instrument of this type with quartz envelope multipliers can go toward the complete analysis of aluminum, the following facts should be noted. With the multisource unit adjusted with sixty microfarads capacity, 480 microhenries inductance, and 50 ohms resistance, it should be possible, taking the photographic sensitivity, as given in a previous publication,[5] to predict as follows: Copper, silicon, magnesium, manganese, and nickel should be determinable with a direct reading instrument down to the 0.001–0.01 percent range. Iron, tin,

[5] M. F. Hasler and J. W. Kemp, J. Opt. Soc. Am. **34**, 21–32 (1944).

zinc, molybdenum, and lead to the 0.01–0.1 percent range. Not quite all of these can be determined simultaneously using the most sensitive lines owing to space limitations at the spectrum. However, since many other lines can be used at higher percentages, it may be possible to arrange conditions so that all these determinations could be done simultaneously in commercial alloys.

In discussing "things to come," the analysis of steels by the direct reading method is an interesting case. Starting with what has been accomplished, Fig. 5 shows the working curve for chromium in steel obtained with the direct reading instrument and the spark source. The chromium line at 4254.3A and the iron line at 4337.1A were used. The average deviation on repeat analysis in the 1 percent region is ±0.03 percent, which is quite acceptable at this time. The half-inch rods furnished by the Bureau of Standards were used for this work, being sparked on a Petrey stand. For more accurate analysis in the 2 percent region, another chromium line showing less self-absorption would be preferable. In any event, the direct reading analysis of chromium in steel is an accomplished fact.

The next question is can manganese, nickel, silicon, copper, molybdenum, and vanadium also be measured with a direct reading instrument? Analyzing photographed spectra obtained with the multisource adjusted to 40 microfarads, 480 microhenries, and 50-ohm resistance, which are reasonably arc-like but still capable of good accuracy, it is found that no interfering lines are less than 2A from chromium at 4254.3A, manganese at 4030.7A, copper at 3274.0A, and vanadium at 4379.2A. In the case of silicon at 2881.6A, molybdenum at 2816.2A, and nickel at 3493.0A, an iron line must be measured together with the elements in question. However, with this source the silicon line is equal to the interfering iron line at 0.2 percent, the molybdenum line at 0.1 percent, the nickel line at 0.1 percent. This means that at these levels the background count contributed by the iron line and optical background will be equal to that contributed by the line to be measured. This, in turn, signifies that where the photographic method would give 2 percent inaccuracies, direct reading might give as high as 4 percent. However, a 0.2 percent ± 0.008 percent silicon determination is still better than can be done chemically as is a 0.1 percent ±0.004 percent molybdenum and nickel determination. Thus this discussion indicates clearly that a direct reading steel analysis of all the elements usually measured spectrographically appears entirely possible, providing percentages over 0.1 percent are being considered in general and that suitable multipliers are made available.

It is apparent that direct reading instruments have been considered in detail in this paper. Our present conception is that a complete installation will consist of a spectrometer unit along the lines of the experimental model discussed, a counter unit which will indicate the percentages of the various elements being determined, and a source unit for the excitation of the spectrum of the sample. From our limited experience, it seems that various controlled source units of sufficient power and stability are on the market at this time; thus the main problem is the development and production of the other two units mentioned.

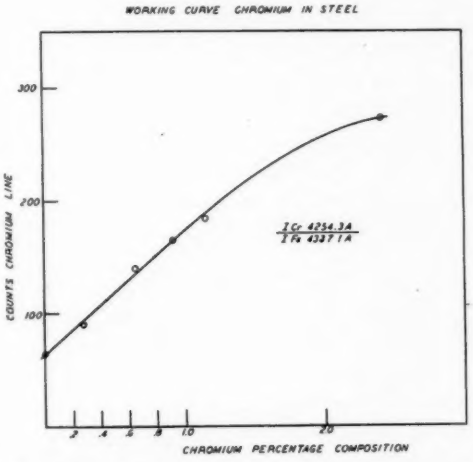

FIG. 5. Working curve for the quantitative determination of chromium in steel, obtained with the direct reading instrument.

With respect to the spectrometer unit, it seems reasonable that the smaller it can be made, the greater its dimensional stability and thus the more accurate the results that it will produce. Since gratings with a small radius of curvature can be produced which will give high dispersion and resolution and since much more room exists for receivers at the focal curve of a grating instru-

ment than a prism instrument, it appears that a short focal length grating instrument will be the most successful type for direct reading.

The counter unit, besides being equipped with twelve counters, will probably house all of the various electrical power packs.

From the foregoing it should be apparent that a practical direct reading instrument is now a reality. It should also be apparent that there are limitations to a set of instruments of this kind which will make them impractical for some time for certain types of spectrochemical testing. General work of a varied nature cannot be handled on the units described as it would be a tremendous chore to search out and identify all of the principal lines of all of the detectable elements. This equipment would also not be feasible for determinations at very low concentrations which now tax photographic instruments to the extreme. However, this last limitation will gradually be eliminated as more stable and sensitive direct reading instruments are produced.

Thus it would appear that for the present, this direct reading equipment will be most successfully applied to the analysis of production alloys, where the utmost in sensitivity is not required and where reasonably long runs on the same alloy type can be expected.

We wish to acknowledge the large part that Mr. J. W. Kemp of Applied Research Laboratories has played in the design and development of our experimental, direct reading instrument.

American Association of Spectrographers

A GROUP of spectrographers in the Chicago area who are interested in furthering spectroscopy have formed the American Association of Spectrographers, which meets on the third Tuesday of every month. The program of each meeting involves presentation of a speaker on some phase of spectroscopy, and a general discussion followed by group meetings for discussion of individual problems. The membership of the group is now about fifty. It is suggested that other persons interested in attending the meetings of this Chicago group or being informed of its activities address the Secretary of the Association, Mr. N. F. Gurley, 4645 West Grand Avenue, Chicago, requesting further information.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 34, NUMBER 12 DECEMBER, 1944

Notes on the Problem of Color Harmony and the Geometry of Color Space

With Reference to Articles by Moon and Spencer

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(Received August 31, 1944)

THE following notes were originally intended merely as a series of comments on certain questions raised by the articles on color harmony by Parry Moon and D. E. Spencer,[1] which have appeared in this journal in recent months, all to be put into a Letter to the Editor. Although, perhaps unfortunately, they have grown into something beyond the compass of such a letter, they are still to be taken as more or less random comments on some of the points considered in these articles, rather than as an attempt at any complete discussion of the subject.

For many years, in courses given at Harvard University dealing with the theory of design, we have used an approximation to an exact representation of color space, like the suggested ω space, by adjusting the Munsell solid so as to give a more nearly accurate correlation of value and chroma distances. This has naturally been a pretty rough approximation, and it has been necessary to make final adjustments visually. I have long felt the need for a more accurate color solid;[2] it would be one in which distances in all directions correspond to actual color contrasts. If the ω space of Moon and Spencer is even near to accuracy in this respect, it seems to me of the greatest value—provided it is used with discrimination.

The authors suggest many interesting theoretical possibilities in the way of achieving color harmony. While certain of these are well known, others are new. It will probably take some time to determine which of these have visual and hence aesthetic significance. I have found by bitter experience that one has to guard against being misled by what might be called one's theoretical desires; and it is necessary to be sure that the satisfaction one may feel in an arrangement based on some theoretical assumption does not turn out to be caused by the working of some quite different and unsuspected principle. While much of the work of Moon and Spencer seems to me perfectly sound, they make some statements which cannot be allowed to pass without question. As frequently happens, I believe, when scientifically trained persons get involved in questions of aesthetics, they tend to simplify somewhat unduly.

My chief concern at the moment is with certain assumptions contained in these articles which seem to me definitely incorrect, and others which are at least open to question.

AESTHETIC MEASURE

First of all, on page 235 in the article, "Aesthetic measure applied to color harmony,"[3] I find the following statement: "Elements of order are identity, similarity, and contrast of hue, value, and chroma." Now by no possible chance, and regardless of Birkhoff or other mistaken authority, can contrast be properly considered an element of order. It is an element of disorder (diversity): for, if order means anything at all, it means likeness or correspondence, or, in other words, uniformity of some kind. It may be uniformity of simple likeness; or uniformity in a gradation or alternation (sequence); or uniformity in opposition (balance). To be sure, contrast, like other kinds of unlikeness or diversity, may be a valuable—indeed necessary—factor in design, in giving interest and hence making the orderly elements of a composition emotionally appreciable, for practically all aesthetic significance depends on a relation between order and disorder; but contrast means unlikeness, or disorder, and it is not an element of order.

[1] P. Moon and D. E. Spencer, "Geometric formulation of classical color harmony," J. Opt. Soc. Am. **34**, 46 (1944); "Area in color harmony," J. Opt. Soc. Am. **34**, 93 (1944); "Aesthetic measure applied to color harmony," J. Opt. Soc. Am. **34**, 234 (1944).

[2] See A. Pope, *An Introduction to the Language of Drawing and Painting, Vol. I, The Painter's Terms* (Harvard University Press, Cambridge, 1929), especially Chapters I–III.

[3] Based on G. D. Birkhoff, *Aesthetic Measure* (Harvard University Press, Cambridge, 1933).

If this is true, it would seem to nullify much of the mathematical calculation in the rest of the article, as well as much of the discussion in the earlier article, "Geometric formulation of classical color harmony." I believe it does.

But, in any case, isn't it better to consider any such formula as Birkhoff's $M=O/C$ (aesthetic measure equals order divided by complexity) simply as a symbol of the principle on which aesthetic experience and aesthetic value work, rather than as something which can be applied literally in the actual practice of design? Birkhoff, in the exposition of his formula, fortunately keeps for the most part at a sub-aesthetic level, and his discussion of the relative value of different polygons—though even here there is plenty of room for disagreement—does well enough to illustrate his point. When he attempts to go beyond this, he is, as Professor Greene has pointed out,[4] on very dangerous ground. When it comes to extended application to anything like color harmony, even in the case of the simplest pattern, there are so many intangible factors in the way of particular circumstance, of individual or group prejudices due to association, tradition, and other causes—and these even under laboratory conditions can never be left entirely out of account—that any exact mathematical calculation is impossible. In an arrangement of any complexity the purely formal factors (even if one tried to ignore the questions of circumstance, association, and so on) become so multiple that any attempt at mathematical calculation is completely useless. And yet, if it is correct, kept in its proper place as a symbol, such a formula may still be useful.

It may be pointed out, however, that as a matter of fact Birkhoff's formula is itself open to serious question, for it is actually a measure of density (or perfection) only, and total aesthetic value would seem to involve also the quantity of order as well as density. In other words, total aesthetic value might be considered a function both of density and of the quantity of diverse elements which have been brought into appreciable orderly relationship.* As T. M. Greene puts it, though in a somewhat negative fashion, "One way of measuring artistic achievement is in terms of how much diversity, discord, or inner conflict has been successfully welded into a state of formal integration to produce a rich synthetic effect."[5] Or, as I have expressed it[2] (with a slight revision), aesthetic experience is a question of reaction to emotionally appreciable order, and the relative value of different aesthetic experiences depends on the quantity and perfection (density) of this appreciable order.[6] This does not detract from the value of Birkhoff's formula, as far as it goes; but it is well to keep in mind that, as a measure of aesthetic value, it perhaps has its limitations.

This is possibly beside the point at the present moment, and this is no place for a full discussion of the problem; only, Moon and Spencer seem to have accepted Birkhoff as final authority, and it must be suggested that the dazzling effect produced by the ability to put philosophical ideas into mathematical terms is no guarantee of accuracy of thought.

COLOR HARMONY IN GENERAL

As to some of the other considerations dealt with in these articles, I can quite understand the authors' dissatisfaction with what they call "classical color harmony," for most of the discussion of color harmony in the past has been confined to the matter of hue interval, with little but a vague regard for value and chroma, to say nothing of area, or what I have always called "quantity." Often, on the analogy of musical interval, there has been an assumption that there must be some sort of virtue in certain specific hue intervals. There is of course no physical basis for hue interval that at all corresponds to that for pitch interval in music; in my opinion there is no visual significance in any particular hue interval, aside from other considerations.* Since color

[4] T. M. Greene, *The Arts and the Art of Criticism* (Princeton University Press, Princeton, 1940), p. 89n.

* This is probably the reason why some of the polygons, which Birkhoff rates as less dense in their order than the square or the equilateral triangle, seem to me at least more interesting. In other words, one has to assume a subaesthetic attitude to come to Birkhoff's conclusions.

[5] Reference 4, p. 218.

[6] Cf. also A. Pope, *A Quantitative Theory of Aesthetic Values, Art Studies* (Harvard University Press, Cambridge, 1925).

* That in color relations we are not dealing with anything approaching the exact intervals of musical harmony is illustrated by the fact that a "good" color arrangement will look well (within certain limits, of course) in a considerable variation of illuminations. Although one illumination may be best, with slight variations of interval produced by other illuminations there is nothing like the discord which would be produced by such variations in the case of sound.

harmony is a spatial affair, it works on an entirely different principle from musical harmony. But Moon and Spencer unfortunately seem not to have got entirely away from the idea of a certain virtue in special color intervals.

What we actually have in color relations is the limiting possibility of complete relationship of all factors (value, hue, chroma), that is, of monotony, on the one hand; and the possibility of extreme variation without any appreciable relationship, on the other. In the first case, that of a single color, there is no aesthetic significance beyond something like occasional delight in the sight of a "bright" or "rich" bit of color, as opposed perhaps surprisingly at the moment to a dull or drab environment. Even in such cases relationship is usually involved. There is a comparison with environment, or one speaks of the beauty or prettiness of a color, thinking of it as "nice" for a dress or a necktie, and then for a particular occasion. There is certainly little significance to be found in the spread of a uniform color over the whole visual area—that is, in complete monotony. And there is, of course, no significance in colors completely unrelated.

Now areas unrelated in color may be brought into relationship by changing their colors in many different ways. They may be brought into a relationship of a sort merely by reducing the contrasts between the colors, without necessarily changing their relation to each other. This means bringing them into some region within color space so that a common element of a dominant quality, like lightness, or darkness, or blueness, or grayness, may give them a certain likeness. Or the sizes of the areas may be changed so that they all exert an equal attraction on the eye, or perhaps an approximation to this near enough to be felt. Or, of course, they may be brought into relationship by making them all the same value, or hue, or chroma, or an approximation to any one or two of these, so long as under the particular circumstances it is near enough to be definitely and sufficiently felt. Or they may be given a uniformity according to Ostwald's idea of isotones, isotints, etc., so far as these are visually significant.[7] Just how much organized relationship there must be and how much diversity must be retained, to give a feeling of delight or satisfaction in the result, depends, I believe, always on the particular circumstances; and it must be remembered that the set-up of colored areas on a neutral ground in the laboratory is a very special circumstance from which it is dangerous to draw general conclusions.

I cannot help being skeptical, therefore, about the significance of the authors' concept of ambiguity. Painters, both in the manner of representation and of pattern (pure design), are constantly dealing with subtle distinctions of color which would fall in the authors' field of first ambiguity. Of course in a given pattern, in which it is necessary to define the shape of a certain area clearly, the contrast of color at the edges of the field must be strong enough to avoid confusion; but often such definition is not desired. I can also understand that a lack of clear distinction might be disturbing in the laboratory set-up; but again this is a special circumstance.

As to the field of second ambiguity, there is, as suggested above, a traditional though probably mistaken belief, caused largely by the idea of analogy with music, that there are such things as discords and chords in hue relations, and there

[7] I am a little doubtful as to the significance of some of these—isotints for example; but, in some circumstances at least, some have, I believe, definite value. It may be of interest that in *The Painter's Terms* (reference 2) I suggested that much of the harmony of color in the later painting of Monet and Renoir and in that of Dodge MacKnight is attributable to the use of colors all between "full color" and white—uniform "isotones"—or approximately that. The uniformity of the "shadow series" is what occurs under ordinary circumstances as objects model from light into shadow [cf. A. Pope, *Introduction to the Language of Drawing and Painting, Vol. II, The Painter's Modes of Expression* (Harvard University Press, Cambridge, 1931), pp. 11–15]. On account of the limitation of "black" paint this can be represented in painting only in a relative fashion. The observation of the principle involved in this, however, is responsible for the illusion of air and space in paintings like those of Van Eyck and Vermeer. The "shadow series" of Ostwald seems to me identical with saturation as the term is sometimes used (and very naturally thought of) even by scientific writers. Luckiesh, for instance, writes: "By the admixture of black to a color (in effect the same as reducing the intensity of illumination) the brightness is diminished without altering either the hue or the saturation." [Luckiesh, *Color and Its Application* (D. Van Nostrand Company, Inc., New York, 1915), p. 80; also cf. Morton C. Bradley, Jr., *Systems of Color Classification, Technical Studies VI*, pp. 240–276 (1938); also cf. A. Pope, *The Painter's Terms*, pp. 50–58.] Many years ago, in a paper I read at a meeting of the Optical Society of America, I tried to point out the significance of this factor, as distinguished from chroma, and also of what I called, for want of a better word, "total brightness"—the equivalent of Ostwald's "isotone"; but there seemed to be no one at the time who could understand what I was talking about.

has been a tendency to attribute particular virtues, especially in the case of two colors, to certain hue intervals, and lack of virtue to other intervals, regardless of other considerations. In this case one has to discount prejudices caused by association and tradition—one might almost say superstitions—some of which are well-nigh universal in particular communities at particular times. Untrained observers, and unfortunately many who have been taught fashionable color theories, are especially susceptible to such prejudices. Only the other day I heard a lady exclaim at the "horrible" effect of a combination of "lavender" mallow and "scarlet" poppy which seemed to me quite delightful—prejudice, too, perhaps, but there was a uniformity of value. And think of the journalistic comments that were once handed out on some of Turner's paintings because they reminded the critic of a Spanish omelette, or possibly included purples which were taboo in landscape painting in the early nineteenth century.

The main point is simply that contrast, and hence unlikeness, gradually increases with greater distance in color space. Colors which contrast strongly in hue, especially if there is also strong contrast in value, and no uniformity in chroma and no proper adjustment of areas, will naturally seem unrelated; but with adjustment of value, or chroma, or area, colors at any hue interval may be given some feeling of relationship and resulting harmony.

But, while this attribution of necessary harmony or discord to certain hue intervals is not unusual, and while a similar virtue has also been attributed to equal intervals, especially of value, this is the first time that I have come on the suggestion that such virtue could be attached to specific value intervals and a lack of it to others! Certainly the examples given on pages 58 and 59 are not very convincing evidence. In the case of achromatic colors (or grays) I am sure that the attribution of ambiguity as a negative factor to certain value intervals is mistaken; and I believe the same thing is true for hue and chroma relations. In all cases it seems to be a question of circumstance how much likeness of various kinds, and how much contrast, or diversity, is required to make the particular result satisfactory.

It seems to me, likewise, that Moon and Spencer tend to put too much stress on the possibility of equally spaced intervals in ω space. As a rule, I believe that uniform intervals of this sort have little visual significance, unless the spatial arrangement is such as to give either a definite feeling of gradual sequence, or a feeling of equal contrast with a background color or with adjacent colors.* In a merely scattered arrangement of juxtaposed areas the uniformity of the intervals does not usually seem very significant, though, in any selection of colors on such a principle, there is likely to be a limitation or uniformity in some other factor to produce a feeling of harmony of another kind.

AREA

It is a distinct relief to find the question of area (or quantity) given proper emphasis, even though the analysis of Moon and Spencer seems to me not always correct. Some years ago I discovered the significance of what I have called "uniformity of attraction" involving this question of area.[2] If equal areas of unrelated colors are arranged against a background, because of the different contrast of each color with the background color, the areas will exert unequal attractions on the eye; there will be a feeling of uneven surface, since some will seem to recede and others to come forward relatively more strongly. By adjusting the size of the areas, however, they can be made to exert an equal attraction, so that not only will the surface appear even, instead of uneven, but the quality of the individual colors will seem actually to change, with a resulting feeling of harmony instead of disharmony.

I believe it is better to think of this as uniformity of attraction rather than as balance (the term used by Moon and Spencer), since, from a visual point of view, balance is more properly applied to instances where there is equal opposition on either side of a vertical axis, or around a center. One may have uniformity of attraction in

* A few years ago one of my students took four colors found by plotting a tetrahedron in color space based on a modification of the Munsell solid with further visual correction. Using these colors in a pattern, regardless of their order, the contrast at the edges of adjacent areas was always the same. The result was visually significant, but not very exciting. By other devices much the same effect may be produced with a result less monotonous.

a composition without reference to an axis or a center, as one may also have balance of attractions on either side of an axis without general uniformity of attraction. Of course, mere terminology has nothing to do with the ideas involved unless it is misleading; but in this case the use of the term balance is, I believe, somewhat confusing, since there has been a mistaken tendency to attribute aesthetic significance to a balance on opposite sides of the vertical axis of color space. It is often forgotten that color space is merely a convenient symbolic representation of the relations of colors which actually exist as areas juxtaposed in two-dimensional space, and that geometric significance in the symbolic representation does not necessarily mean visual significance.

The working of the principle of uniform attraction is observed most strikingly perhaps when several colored areas are placed on a black background. The areas nearest black in color space are naturally larger, and those farther away progressively smaller. We have found exercises in adjusting the size of areas on grounds of various colors, as well as black and white, very useful in training the eye in sensitiveness of judgment. Uniformity of attraction may of course be achieved by eye without recourse to mathematical calculation.

In the above instances it is assumed that the background color is seen on all sides of each area. What Moon and Spencer have failed to take into account is that the minute the areas are juxtaposed, the relative sizes of the areas have to be changed, for the color of each area is no longer contrasted solely with the background color, but, along certain sections of the edge of each area, it is contrasted with one or more other colors. It is not a matter merely of adaptation to a background color or to an average adaptation point, but a question of contrast of each color with *adjacent* colors. Change the spatial arrangement of areas in any way and the sizes of the areas have to be changed. It is the contrast at the edge of the area that, together with its quantity, determines the attraction.

It is evident, therefore, that in an arrangement in which there is no background but all areas are juxtaposed—that is, each color contrasted at the edge of its area not with one but perhaps with several other colors, all varying in hue, value, and chroma—the problem becomes so complicated that mathematical tables can hardly be of much use.

This is the difficulty which Moon and Spencer have run into in the design in Fig. 4 (page 102).[1] I defy anyone (at least without prejudicial desire) to feel any organization of color relations in this design, aside from the fact that the colors are all uniformly achromatic, or gray. No account has been taken of the fact that the colors vary in attraction according to the contrasts at the edges. *N*8, for example, in a large part of its extent, comes against *N*9, and this affords a very weak contrast. On the other hand, the area of *N*8 in the lips, placed against *N*6 and *N*3, affords a strong contrast, and it exerts a much greater attraction on the eye than the same sized area of *N*8 in any other part of the design. To achieve a proper mathematical solution, the contrast along every edge would have to enter into the calculation. By visual discrimination it would of course be fairly simple to achieve a satisfactory result (allowing for the rather atrocious character of the spatial composition); it would be very different from the result obtained in Fig. 4.

The point I want to make is that an understanding of the principle involved may be of real use, and that the concept of ω space can be helpful in making this principle clear. But the multiplicity of factors involved in the application to specific design problems is likely to make mathematical calculation not only hopelessly involved but, if some of the factors are ignored, definitely misleading.

And these are not all the complications by any means. For example, to achieve uniform attraction in the case of a number of areas, an outline of another color may be used to change the attraction of a given area; or accents of dots or of other pattern may achieve the same result. Try calculation in this case!

In addition to the above, when it comes to the principle, or law, involved, I question the accuracy of the authors' premises in regard to *scalar moment*. This is defined as the product of the area of the patch and the distance in ω space between the adaptation point and the color point.

Bradley[8] has suggested that in order to achieve equal attraction in the case of two colors placed on a background, the areas must be inversely proportional to the cube of the contrasts with the background color (distance in color space). This seems to come at least near to the proper relation necessary, though it may not be exact, and it has been suggested that a logarithmic progression may be involved. At any rate, Bradley's formula has proved useful as a starting point for visual discrimination; for it has been found that beginners have difficulty in making the difference in the size of areas sufficiently great. Moon and Spencer make the size of the areas depend on the exact contrast (exact distance in ω space). Although geometrically one would expect this, we have found it not to work in actual practice.

In conclusion, I may say that while Moon and Spencer in their concept of ω space suggest important possibilities for the formulation of general principles underlying color harmony, they have tended, I believe, to oversimplify a very complex subject, and, what is natural enough in a way, have leaned too much toward possibilities of geometrical organization without considering sufficiently the matter of visual significance. They might possibly have given more attention to work that has already been done along somewhat the same line. There are many other questions which might be considered; but I think I have pointed out some of the principal difficulties, and I have no intention here of entering into a complete discussion of the subject. The advantages of an exact representation of color space lies in the fact that by its means we can determine more definitely the underlying principles involved in color harmony. There are many different possibilities in the way of significant organization of color relations. Some of these we understand fairly well, others less definitely, while I believe there may be still others which we know little or nothing about from a theoretical point of view. But each kind of relationship is a possibility which may or may not be involved in a particular design. In the discussion of color harmony in the past there has been too much of a tendency to lay down rules which must be followed. There are no rules; there are only possibilities.

[8] Morton C. Bradley, Jr., *A Theory of Tone Attraction, Technical Studies* (Fogg Museum of Art, Cambridge, 1933), Vol. II, p. 3.

PRACTICAL CONSIDERATIONS IN THE REPRESENTATION OF COLOR SPACE

On the general subject of the geometric representation of color space, I should like to add that what I have called a "working tone (or color) solid"[2] has proved very satisfactory in actual practice in painting and design for the last thirty years and more. In spite of its obvious inaccuracy, it is the simplest possible device to use in giving a student a concept of the three dimensions of color—hue, value, and chroma—and even of related factors like Ostwald's "isotones" and "shadow series." Horizontal planes indicate uniformities of value; vertical planes, radiating from the vertical axis, uniformities of hue; and concentric cylinders about the axis, approximate uniformities of chroma. Moreover, it suggests a proper correlation of vertical (value) and horizontal (chroma) distances, and the student can make the basic scales of value, hue, and chroma with considerable accuracy with the use of simple pigments which are employed in actual painting. He thus learns to think and visualize in terms of color relations very quickly.

With an explanation of the difference between additive and subtractive mixing, the limitations of the working solid are easily understood. For more accurate results, it is perfectly easy to shift, when necessary, to the Munsell solid (as shown in the Munsell *Book of Color*); we have done this constantly, although even here allowances for inaccuracies have had to be made. It would be just as easy to shift to the ω space suggested by Moon and Spencer. Moreover, in spite of the accidental nature of pigment mixing, the results of the mixtures of the usual pigments employed in painting are represented with a fair degree of accuracy by lines drawn through the working solid. The allowances necessary are easily made; it is only occasionally that they are very great. The distances between points in the solid also represent with fair accuracy the contrasts between the colors represented by these points; and the various possibilities in the way of harmony of value, hue, and chroma relations can be visualized with reference to the whole solid, or can be shown graphically in two-dimensional diagrams, like the circular plan for hue and chroma, and the vertical elevation for value and chroma. Where so many final adjustments

have to be made by visual discrimination in any case, the working solid is more useful than a more exact one, except in special cases. This is particularly true for the actual practice of painting where a more exact arrangement, like that of Munsell, is of little use.*

* It may be noted that in courses in camouflage given at Harvard University during the last two or three years, it was found that the working solid was much more convenient for both teaching and practice purposes than the Munsell solid the use of which was also tried out. With pigments of low chroma, such as are used in camouflage, mixtures of colors fall almost exactly on the lines drawn through the solid.

It has been suggested that the use of such a working color solid could be justified on the analogy of the practical advantage of the tempered scale of the pianoforte. To be sure, the tempering is in this case rather violent; but it has its practical advantages. As a representation of color space, and as an instrument for thinking in color terms, the distortions are not nearly as great as in the Ostwald solid. The great disadvantage of the Ostwald solid is that geometrical distances have no relation to color contrasts.

Letter to the Editor

Reply to Arthur Pope

PARRY MOON, *Massachusetts Institute of Technology, Cambridge, Massachusetts*
AND
DOMINA EBERLE SPENCER, *Tufts College, Medford, Massachusetts*
October 9, 1944

WE are indebted to Professor Pope for his careful analysis of our papers on color harmony. We agree that analogy between color and music is fallacious and likely to lead to all sorts of errors. No such analogy is implied in our work. We also agree that color harmony is an extremely complex subject. On some points, however, our opinions differ.

Professor Pope objects to our use of "contrast" as an "element of order." Is this not perhaps merely a semantic difficulty? He considers the word "contrast" to be synonymous with "diversity" and therefore the antithesis of "order" or "uniformity," while we have used the expression "element of order" in the specialized sense of Birkhoff. There are relations among colors which tend to make a pleasing design. We have called all such relations *elements of order*, irrespective of whether they cause a pleasing sense of similarity or a pleasing sense of contrast. In obtaining the aesthetic measure, therefore, one adds a positive numerical factor for each similarity of hue, value, and chroma and for each contrast of hue, value, and chroma. Certainly, one should not subtract the latter just because the word "order" does not seem to agree with the word "contrast." The formulas and numerical values are independent of the words that may be used in discussing them.

"Ambiguity" is another word whose meaning appears most clearly by actual use of the system. We agree that the artist may employ any hue intervals he wishes: As stated in the first paper, *no intervals are forbidden*. But some intervals give higher aesthetic measure than others. Surely there is a region of ambiguity between identity and similarity, where the observer has a feeling or uncertainty that tends to lessen the aesthetic effect of the design. Our specification of regions of ambiguity does not mean that these regions are forbidden—their use merely tends to produce lower values of aesthetic measure. It is true that subtle variations in color occur in nature and are often very beautiful, and similar gradations are produced by the artist. But, as specifically noted in our writings, such refinements are beyond our theory of color harmony, which deals with areas of uniform color.

The treatment of *area* is an example of the idea that a simple formulation, even though imperfect, is better than no formulation. We have assumed that the eyes are adapted to a neutral background of Munsell value 5 and that they travel over the colored design with sufficient speed so that the state of adaptation remains practically fixed. The complexity associated with the difference in contrast between all the contiguous parts of the design is ignored. Admittedly, this is a highly idealized assumption. But it allows simple mathematical expressions that appear to approximate the truth. Perhaps a more comprehensive formulation can be found in the future. Also, the theory does not pretend to cover the choice of hues for particular uses or the effects of tradition and habit. All these are subjects for further research.

Professor Pope feels that color harmony is too complex a subject to allow mathematical treatment. We are more optimistic. A landscape or portrait painter will probably not find calculations desirable or necessary, though even he may find the results of our theory useful in a semiquantitative sense. Our aim, however, was primarily to develop a tool that could be employed even by a person of very limited artistic training. The theory has quantitative significance particularly in such subjects as poster design, interior decoration, fabric design, and dress. We trust that in these fields the present theory will be of practical benefit

Proceedings of the Twenty-Ninth Annual Meeting of the Optical Society of America

HOTEL PENNSYLVANIA, NEW YORK, NEW YORK, OCTOBER 20 AND 21, 1944

THE twenty-ninth Annual Meeting was, in spite of the many wartime restrictions, one of the most successful in the history of the Society. A total of 418 members and guests registered for the several sessions.

SYMPOSIUM OF INVITED PAPERS

Friday, October 20, was devoted to a symposium of eight invited papers under the general title "A Review of Some Applications of Optics."

Mr. Frank Benford, of the General Electric Company, opened the morning session with a paper on "The Projection of Light." Optically, the fundamental difference between a searchlight and other image-forming systems, such as the camera lens, lies in the fact that the latter have a unified optical surface and a differentiated image; whereas the former has a differentiated optical surface and a quasi-unified image. The distribution of light in a searchlight beam is computed by determining the image formed by a small section of the mirror or lens. The zonal images may then be summed graphically to obtain the complete beam. Some of the more important characteristics of the projected beam were discussed, such as angular width and applicability of the inverse square rule of illumination. Test data on a mirror made of a combination of ellipsoid and hyperboloid were presented to show how the filament image can be suppressed.

"Practical Applications of Metallic and Non-Metallic Films on Optical Elements" were reviewed by Dr. Dean A. Lyon, of Washington, D.C., who emphasized the increasing importance of thin films as coatings on optical surfaces. Some of the properties of films of silver, aluminum, and rhodium were discussed. Among the non-metals, it was shown that zinc sulfide is useful as a beam splitter, and that magnesium fluoride is especially suited for antireflection coatings.

Professor Maurice Ewing, of the Wood's Hole Oceanographic Institution, conducted a tour of the depths of the sea in his paper "Photography of the Ocean Bottom." In the years 1940 to 1942, the author and his co-workers developed cameras and techniques for use in all depths of water, and have obtained good photographs of the bottom in depths of 500 to 600 fathoms and some photographs at 2700 fathoms. Details of the apparatus and methods of handling the camera were described. The photographs assist biologists, geologists, and oceanographers, aside from their more obvious usefulness in the study of submerged wrecks.

Dr. Robert D. Heidenreich, of The Dow Chemical Company, spoke on "Techniques in Applied Electron Microscopy." Some of the characteristics of the images formed by the electron microscope were treated, including resolution, intensity, and diffraction at the straight edge. Of interest was the comparison between optical and electron photomicrographs, the latter having been obtained by the author's polystyrene-silica method. The usefulness of the electron microscope is considerably enhanced by the employment of stereoscopic methods. As an example, the quantitative determination of the contours of diffraction gratings was given. Several examples of the method were illustrated by the use of Polaroid vectograph slides, using a special projection screen and Polaroid viewers which were made available by the Polaroid Corporation, Cambridge, Massachusetts.

The afternoon session began with a paper by Dr. Norbert J. Kreidl, of the Bausch & Lomb Optical Company, entitled "Recent Studies on the Fluorescence of Glass." Changes in the wave-length and intensity of the fluorescence of indicators can be correlated with changes in the ionic structure of glasses. Uranium and europium are suitable indicators for intensity studies, divalent manganese for wave-length studies. The intensity of fluorescence is increased and the wave-length decreased by a high percentage of the more strongly bonded "glass-forming" ions like silicon, which unite with oxygen to form the basic network characteristic of the glassy state. As more weakly bonded "modifier" ions, e.g., sodium, enter and distort the basic network, the green

fluorescence of a glass containing divalent manganese shifts toward longer wave-lengths. The various types of optical glass can be distinguished by their characteristic fluorescence in ultraviolet. Light flints fluoresce a brilliant blue; heavy flints a dull green; and the borosilicate crowns, spectacle crowns, and barium crowns are characterized by white to purple fluorescence. A shop apparatus for identification work was demonstrated.

"Color Motion Pictures of the Whole Sky" was presented by Mr. H. R. Condit, of the Eastman Kodak Company. Time lapse pictures of the whole sky from sunrise to sunset had been taken over a period of several years by means of a motor-driven camera which photographs onto one 16-mm frame of Kodachrome film the virtual image formed by a convex mirror. The pictures showed the sky on different days, taken at one, four, or ten frames per minute. They are useful in studying the formation and movement of clouds. A record of the integrated flux from the whole sky is kept by means of a continuous automatic photometer.

Mr. P. R. Irish, of the Bethlehem Steel Company, spoke on "The Application of Spectrochemical Analysis in the Steel Mill." The discussion covered some of the factors that must be considered if rapid large-volume routine analyses are to be made with an acceptable accuracy. Among these are control of the photographic plate characteristics, selection of line pairs, and construction and adjustment of the interrupted spark. A precision of a standard deviation of 2 percent to 3 percent of the content is obtained under conditions involving the use of 7000 separate spectrograms a week which are exposed on 1300 plates. While a precision of this order is acceptable for concentrations up to 3 percent and 4 percent, an improvement in the method is necessary if spectrochemical analysis is to be extended to concentrations of 10 percent and 20 percent with entirely satisfactory results.

The symposium concluded with a paper on "Some Techniques for Securing and Maintaining Effective Night Vision" by Professor Walter R. Miles, of Yale University. The basis for using red goggles for producing dark adaptation was reexamined in terms of new experimental results. Differential monocular adaptation for 30 minutes with red and blue filters matched for luminous transmission was compared with the opaque shielded eye. Threshholds following red adaptation were found to fall only slightly above those for the dark-adapted eye. On the other hand, the threshholds for the blue-adapted eye fall considerably above those for the red.

LUNCHEON MEETING

The Luncheon Meeting on Friday was attended by 169 guests. President Pfund opened the meeting with a tribute to the memory of Mr. Edward Bausch, who had done so much for the advancement of optics. He then introduced Professor Richard M. Sutton, who gave a stimulating resume of a meeting held in May, 1944, under the auspices of the National Research Council. A full report of this meeting appeared in *The Review of Scientific Instruments*.*

ANNUAL BANQUET

There were 314 members and guests at the Annual Banquet on Friday evening. At the conclusion of the meal, the Adolph Lomb Medal was awarded to Dr. R. Clark Jones, after which Professor R. W. Wood discussed some "Reminiscences."

AWARD OF THE ADOLPH LOMB MEDAL

Before bestowing the Medal, President Pfund called upon Mr. Frank Benford for the report of the Medal Committee. The citation read as follows:

Dr. R. Clark Jones, the Optical Society has awarded you the Adolph Lomb Medal for your published work entitled "A New Calculus for the Treatment of Optical Systems." In these papers you courageously attack an optical problem of great complexity, and in your solution you have contributed a general method that may find many applications. Since the problem you set yourself is one of many variables, such as polarization, rotation of the plane of polarization, phase relationships, and absorption, it seems entirely appropriate that you used complex numbers and the notation of the matrix algebra as your weapons of attack.

It took scholarship to understand the full nature of the problem, and mathematical ability of a high order to formulate the equations. But above all, it took optimism to believe it could be done, and you have demonstrated that you amply possess all these essential qualities. With this Award go the thanks and the best wishes of the Optical Society of America.

* Rev. Sci. Inst. **15**, 283 (1944).

PROFESSOR R. W. WOOD, HONORARY MEMBER

It was with much pleasure that President Pfund then announced the election of Professor R. W. Wood as an Honorary Member of the Optical Society. Professor Wood graciously accepted and then delighted his audience, not only with an enjoyable hour of reminiscences, but also with souvenir replicas from the hand of the master himself.

BUSINESS SESSION

A business session was held on Friday afternoon, President Pfund presiding. The reports of the Secretary, Treasurer, and Editor of Publications were presented and accepted.

CONTRIBUTED PAPERS

The abstracts of the contributed papers are appended.

MEETING OF THE BOARD OF DIRECTORS

A meeting of the Board of Directors was held on Thursday, October 19. The minutes of this meeting appear on page 777 of this issue.

PROGRAM OF SESSIONS

(TITLES AND ABSTRACTS OF PAPERS)

Symposium of Invited Papers

A REVIEW OF SOME APPLICATIONS OF OPTICS

1. **The Projection of Light.** FRANK BENFORD.
2. **Practical Applications of Metallic and Non-Metallic Films on Optical Elements.** DEAN A. LYON.
3. **Photography of the Ocean Bottom.** MAURICE EWING.
4. **Techniques in Applied Electron Microscopy.** ROBERT D. HEIDENREICH. (With stereoscopic slides.)
5. **Recent Studies on the Fluorescence of Glass.** N. J. KREIDL.
6. **Color Motion Pictures of the Whole Sky.** H. R. CONDIT.
7. **The Application of Spectrochemical Analysis in the Steel Mill.** P. R. IRISH.
8. **Some Techniques for Securing and Maintaining Effective Night Vision.** WALTER R. MILES.

Contributed Papers on Vision and Color

9. **The Relation of Protanopic to Normal Vision.** DEANE B. JUDD, *National Bureau of Standards.*

It has been shown[1] that the characteristics of average protanopic vision can be represented by functions K and W_P connected to those (X, Y, Z) defining the 1931 I.C.I. standard observer by the linear relations:

$$\left.\begin{aligned} K &= Z, \\ W_P &= -0.460X + 1.359Y + 0.101Z, \end{aligned}\right\} \qquad (1)$$

where W_P is the protanopic luminosity function. From this luminosity function it is possible to find by the usual methods of computation how much darker to an average protanope are normally reddish colors. If the colors are those of surfaces it is customary, for normal vision, to estimate lightness or darkness from the normal luminous reflectance of the surface for I.C.I. Illuminant C relative to magnesium oxide (Y/Y_0), where for magnesium oxide $X_0 = 0.9804$, $Y_0 = 1.0000$, and $Z_0 = 1.1812$. For protanopic vision we take instead the relative protanopic luminous reflectance $W_P/(W_P)_0$, which from (1) is equal to $W_P/1.0274$. If Q be the ratio of the protanopic luminous reflectance of a surface to its normal luminous reflectance, we may write: $Q = W_P/1.0274\,Y$, whence from (1) it is found that:

$$x = y(Q - 1.224)/(-0.546) + 0.180. \qquad (2)$$

Equation (2) represents a family of straight lines on the (x, y) chromaticity diagram passing through the point $(x = 0.180,\ y = 0.000)$, whose slopes are determined by the corresponding values of Q. For $Q = 1$, the line passes through the point representing illuminant C. For $Q = 2$, the line

passes close to 450 mμ; all surfaces whose chromaticities are represented on this line have relative protanopic luminous reflectances equal to twice their normal reflectances. For $Q=0.10$, the line passes close to 650 mμ; on this line relative protanopic luminous reflectances are 10 percent of normal. For $Q=0$, Eq. (1) defines the protanopic alychne, or lightless line, analogous to the normal alychne, $y=0$, which results from setting Q indefinitely large. By adding this family of straight lines to those indicating the chromaticity confusions of an average protanope,[2] it is possible by means of the (x, y) chromaticity diagram to give a complete and concise summary of the relation between protanopic and normal vision.

[1] D. B. Judd, J. Opt. Soc. Am. **33**, 351 (1943).
[2] D. B. Judd, J. Opt. Soc. Am. **33**, 305 (1943).

10. Spectral Sensitivity of the Dark-Adapted Human Eye.* GEORGE WALD, *Biological Laboratories of Harvard University.*

I have constructed a spectral adaptometer which uses as source a high pressure mercury arc. From its radiation, nine spectral regions are isolated with narrow band-pass filters: lines at about 365, 405, 436, 492, 546, and 578 mμ, and lines and narrow continua at 623, 691, and 750 mμ. The intensity at each wave-length is accurately varied with an annular photographic wedge. A variable fixation point permits the test field to be fixated centrally, or at any point within a 10° radius of the fovea. A camera shutter exposes the field for flashes of 1/25 second. With this instrument the absolute thresholds of the wholly dark-adapted eye have been measured centrally and peripherally (8° above the fovea). The peripheral measurements yield a rod sensitivity function (1/threshold), maximal at about 505 mμ. The field appears colorless at all wave-lengths to most observers; about 1/10 the observers report the 691- and 750-mμ radiations as reddish. The rod sensitivity at 365 mμ is 4.2 log units lower, and at 750 mμ is 6.4 log units lower than at the maximum. A 1° field fixated centrally should fall wholly within the rod-free area of the fovea, and so should stimulate only cones. Adequate fixation of this type can be obtained in the wholly dark-adapted eye with relatively untrained subjects. The spectral sensitivity, determined under these conditions, is comparable with previous photopic measurements by flicker or step photometry. The field is usually reported at the threshold as colored at all wave-lengths. The sensitivity is maximal at about 565 mμ, and at both 365 and 750 mμ is 4.2–4.4 log units lower than at the maximum. The absolute sensitivity of the fovea is much lower than that of the periphery below about 670 mμ. At this wave-length the functions cross, and at higher wave-lengths the fovea is more sensitive than the periphery. The foveal function as a whole lies at longer wave-lengths than the standard I.C.I. luminosity curve, and agrees above 510 mμ with more recent photopic measurements.[1] Below 500 mμ, where these previous measurements—including the I.C.I. curve—show a small but distinct inflection when plotted as log luminosity, the present measurements show a large hump of increased sensitivity. Due to this the foveal sensitivity in the violet is as much as a log unit higher than previous measurements of photopic luminosity have indicated.

* This research has been supported in part by the Josiah Macy, Jr., Foundation.
[1] A. Dresler, Das Licht **7**, 81 (1937); P. Jainski, *Dissert. Tech. Hochschule* (Berlin, 1938); N. T. Fedorov *et al.*, J. Phys. U.S.S.R. **3**, 5 (1940).

11. Dark Adaptation Following Light Adaptation to Red and White Lights. SELIG HECHT AND YUN HSIA, *Columbia University.*

The scotopic and photopic luminosity curves differ by about 45 mμ in their positions in the spectrum. Moreover, on a real energy basis they are almost coincident in the red beyond 650 mμ. It therefore follows that when red light and whole white light are made equal in brightness by the photopic(cone) system, the red will be much less effective than the white in stimulating the scotopic (rod) system. This conclusion has been applied by using red light or red goggles to achieve preliminary dark adaptation of rod vision while maintaining cone vision in a relatively high and useful state. Recently this theory and practice have been questioned, and it has become necessary to reinvestigate them. For this purpose we have equated the brightness of white light and red light by means of photopic vision. These lights were then used for light adapting the eye, after which its dark adaptation was measured. In one series six subjects were adapted to a brightness of 30 millilamberts. In all cases there is no question that dark adaptation following red pre-adaptation is faster, and sooner over, than following white pre-adaptation. To reach 0.5 log units above the final rod threshold required only 1.9 minutes after red pre-adaptation as compared to 6.2 minutes after white pre-adaptation. In another series we measured with one subject the influence of the pre-adaptation brightness on the time to reach a fixed threshold level during the subsequent dark adaptation. The relation between time and log I follows a sigmoid curve. The curves for red and white pre-adaptation are parallel, but the red is displaced about a log unit to the right on the log I axis. Therefore, to achieve the same subsequent dark adaptation in the same time one may remain light adapted to about 10 times as high a photopic brightness of red light as of white light.

12. A Mathematical Analysis of Glare. DOMINA EBERLE SPENCER, *Tufts College.*

It is well known that glare is a factor that should be carefully limited in lighting design. But a quantitative study of the problem has been lacking except for fragmentary experimental results.

There are many ways in which the subject can be approached. The paper considers three typical situations, for each of which glare criteria are established: (a) Eyes are fixed on the work. A non-uniform surround usually reduces visual acuity and contrast sensitivity. (b) Eyes are adapted to glare source. The reduction in vision is evaluated for the condition that the eyes are suddenly shifted back to the work. (c) Eyes are adapted to the work and look suddenly at the glare source. An unpleasant sense of shock may be experienced. It is evaluated by means of the

photochemical theory of vision. A comparison of the three methods establishes criteria for the limitation of glare helios in interior lighting. These criteria should be particularly useful in lighting design and should help to place that subject on a quantitative basis.

BIBLIOGRAPHY

L. L. Holladay, J. Opt. Soc. Am. **12**, 306 (1926).
P. G. Nutting, Trans. I. E. S. **11**, 1 (1916).
E. W. Fowler and C. L. Crouch, Illum. Eng. **36**, 897 (1941).
P. Moon and D. E. Spencer, "Visual data applied to lighting design," J. Opt. Soc. Am. **34**, 605 (1944); "A modified photochemical theory of vision," to appear in J. Opt. Soc. Am.

13. Modified Chromatic Value Color Space. J. L. SAUNDERSON AND B. I. MILNER, *The Dow Chemical Company.*

Adams'[1] "chromatic value" diagram has been modified by the equations

$$\zeta_1=(V_x-V_y)(9.37+0.79\cos\theta)$$
$$\zeta_3=(V_x-V_y)(3.33+0.87\sin\theta),$$

where

$$\tan\theta=\frac{0.4(V_z-V_y)}{V_x-V_y}.$$

The Munsell renotations[2] plotted in the $\zeta_1\zeta_3$ plane have very satisfactory chroma spacing, even for the extrapolated colors, for all values. The spacing of hues with respect to θ is fairly uniform, although the greens appear to be somewhat crowded together. There are also varying degrees of curvature to some of the constant hue lines. When the above equations are used with the third equation, $\zeta_2=2V_y$, a three-dimensional space model is obtained. The constants for the three equations have been chosen so that the ζ's are in units of one chroma step, assuming that one value step is equal to two chroma steps. The size of the ratio of units of ζ_2 to ζ_1 and ζ_3 is not in agreement with the ratio of 1:8 found by Bellamy and Newhall,[3] but does agree with the color difference formula of Nickerson,[4] ($1V=2C$) and with the previous conclusions of the authors.[5] The space also gives 1 chroma step = 3.18 hue steps at chroma 5 (compare $1C=1.5H$ of the Nickerson[4] formula) and $1C=2.67H$ at chroma 6 (compare $1C=2.75H$ from Bellamy and Newhall[3]).

[1] E. Q. Adams, J. Opt. Soc. Am. **32**, 168 (1942).
[2] S. M. Newhall, D. Nickerson, and D. B. Judd, J. Opt. Soc. Am. **33**, 385 (1943).
[3] B. R. Bellamy and S. M. Newhall, J. Opt. Soc. Am. **32**, 465 (1942).
[4] Dorothy Nickerson, Textile Research **6**, 505 (1936).
[5] J. L. Saunderson and B. I. Milner, J. Opt. Soc. Am. **34**, 167 (1944).

14. On the Use of Working Standards of Didymium and Vitrolite Glasses for Spectrophotometric Measurements. HARRY J. KEEGAN AND KASSON S. GIBSON, *National Bureau of Standards.*

Calibrated working standards of didymium and of Vitrolite glasses facilitate the accurate measurement of spectral transmittance and of spectral reflectance on recording spectrophotometers at the National Bureau of Standards. A number of Corning 512 and Jena BG 11 didymium glasses and of Vitrolite and Corning Thermometer White glasses have been submitted by governmental and industrial laboratories for their calibration as working standards relative to the NBS standards. The methods employed in the calibration of the NBS standards are explained. Following the practice previously reported[1,2] for the use of certain didymium glasses for wave-length calibration of recording spectrophotometers, direct determinations of the wave-lengths of maximum absorption for slits corresponding to 4, 8, 10, and 20 millimicrons have been made on the Gibson photoelectric spectrophotometer[3] for the visible and near infra-red spectrum for Corning 512 and Jena BG 11 didymium glasses. These determinations have been checked by differences from wave-lengths obtained on two General Electric recording spectrophotometers, one with 4- and 8-mμ slits, and one with 10-mμ slits. The following wave-lengths for the slit widths noted are believed to be applicable to the present stocks of these glasses, although of course this cannot be guaranteed: (1) Corning 512 didymium glass, for 10-mμ slits, 441.$_0$, 475.$_5$, 528.$_7$, 585.$_0$, 684.$_8$, and 743.$_5$ mμ; for 20-mμ slits, 745, 808, 883, and 1067 mμ. (2) Jena BG 11 didymium glasses, for 10-mμ slits, 440.$_6$, 475.$_1$, 526.$_7$, 583.$_0$, 683.$_5$, and 744.$_5$ mμ; for 20-mμ slits, 745, 807, 882, and 1072 mμ. It is believed that the uncertainties of the wave-lengths for the 10-mμ slits do not exceed ±1.0 mμ, and for the 20-mμ slits ±2.0 mμ. The use of a polished Vitrolite glass as a working standard of spectral apparent reflectance relative to freshly prepared MgO was previously reported at a Symposium on Spectrophotometry in the Pulp and Paper Industries.[4] Although the data for the NBS Vitrolite standards of spectral apparent reflectance may not be safely applied to all such glasses, typical curves for different melts and thicknesses both for these and for Corning Thermometer White glasses will be shown together with curves for various calibrations of one NBS Vitrolite and comparative curves of the submitted test glasses and of the standard. The amount of the specular component of the reflected light included in the measurement has, of course, an important effect on these calibrations.

[1] K. S. Gibson and H. J. Keegan, J. Opt. Soc. Am. **28**, 372 (1938).
[2] K. S. Gibson and H. J. Keegan, J. Opt. Soc. Am. **31**, 462 (1941).
[3] K. S. Gibson, "Direct reading photoelectric measurement of spectral transmission," J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 693 (1923).
[4] K. S. Gibson, Tech. Assoc. Papers **23**, 475 (1940). Also published in Paper Trade J. Tech. Sec. **111**, 135 (1940).

15. On the Measurement of the Spectral Apparent Reflectance of Low Reflecting Materials. HARRY J. KEEGAN, *National Bureau of Standards.*

The photometric cam of the General Electric spectrophotometer represents the tangent square law except for the region near zero at which point an infinite radius would be required. The necessary departure of the cam from the law introduces errors in the photometric scale below 1 percent on the normal (0 to 100 percent) cam and below 0.1 percent on the "×5" (0 to 20 percent) cam.[1] Accurate measurements of spectral apparent reflectance less than 0.2 percent, however, may be made on this instrument by the use of procedures designed to bring the curves up on the photometric scale so that the zero curve will be above this region of error. Two such procedures will be illustrated. A specification for reflection-reducing films produced by the evaporation process[2] requires that the minimum reflectance

between 450 and 650 mμ shall not exceed 1.5 percent for borosilicate crown glass nor 0.6 percent for extra dense flint glass. The zero curve in this case was pushed above the region of error by using dark gray paper as the backing instead of a completely non-reflecting cavity. A check on this method may be had for these nearly non-light-absorbing glasses by means of the fact that the reflectance plus the transmission should be nearly unity. The reflectance of many of the standard samples of pigments issued by the National Bureau of Standards falls nearly to zero for some parts of the spectrum. These samples are measured in the form of a wet paste (linseed oil or glycerine vehicle) of the dry paint pigment placed in optical contact with a thin clear piece of microscope cover glass and held in place in the trough of an aluminum holder. The procedure to avoid the region of error on the photometric scale is to include the specular component. The zero correction curve is obtained by measuring the spectral reflectance of the cover glass in contact with a film of the vehicle on a black glass. The particular black glass selected, judged by the reflectance curve for perpendicular incidence,[3] had dispersion and index of refraction as nearly as possible those of the cover glass. Errors which result from neglect of these precautions will be demonstrated not only in the form of spectrophotometric curves but also in terms of the I.C.I. (x, y) chromaticity diagram.

[1] J. L. Michaelson, J. Opt. Soc. Am. **28**, 365 (1938).
[2] O.S. 1357. Specifications for reflection-reducing films produced by the evaporation process, revision A of 23 November 1942. Navy Department, Bureau of Ordnance.
[3] D. B. Judd, J. Research Nat. Bur. Stand. **29**, 329 (1942). (RP 1504.)

16. Thickness Control of Color of Sharp Cut-Off Type of Glasses. H. P. GAGE, *Corning Glass Works.*

It is noted that certain spectrophotometric curves, notably those of the sharp cut-off type of colored glasses, have essentially the same shape—merely displaced in wave-length for different thicknesses. If this were strictly true, it must be that the curve between log log transmission and wave-length is a straight line, since only a straight line can be moved vertically (by change of glass thickness) and match the same curve moved horizontally (by change of wave-length). It has been found by calculation and experiment that, if it is assumed that the log log transmittance versus wave-length curve is to be a straight line, the curve between transmittance and wave-length has an S shape which is not a bad fit for actual spectrophotometric curves of certain glasses. For those glasses for which the theoretical transmission curves fit most perfectly, the following relations also occur:

(a) Symbols used:
T is transmission of glass at a given wave-length.
T max.—Maximum transmission at any wave-length.
T' is T/T max.
t is thickness. λ is wave-length.
(b) The best results obtain when the value $T' = T/T$ max. is used to compensate for both the non-selective surface losses, reflection, and general non-selective body absorption.
(c) If the log log T' straight line is displaced upward by an amount $\Delta \log t$, it is equivalent to a horizontal displacement by an amount $\Delta\lambda$; therefore, for equal spacing of cut-off at any given level, equal changes of $\log t$ must be made.

With a series of infinitely sharp cut-off curves, the values of Y, x, y, z (transmission and the x, y, and z I.C.I. coordinates), may be calculated for any wave-length cut-off as reported for the O.S.A. data in 1931,[2] where the assumption was made that there was complete transmission for all wave-lengths longer than a certain cut-off wave-length, and complete absorption for all shorter than this. The relation of $\Delta y/\Delta\lambda$ can, for example, be empirically plotted against λ. It can be assumed that the change of the y coordinate for fairly sharp cut-off spectra will approximate this same variation in $\Delta y/\Delta\lambda$. From the first assumptions that $\Delta\lambda/\Delta \log t$ can be determined from the shape of the log log curve corresponding to a given glass, it is possible to calculate the required $\Delta \log t$ and hence the change in t (thickness of the glass) to produce any desired change Δy in the y coordinate within the capabilities of the glass.

[1] For properties of log log curve see Hardy, *Handbook of Colorimetry.*
[2] H. P. Gage, J. Opt. Soc. Am. **21**, 433 (1931).

Contributed Papers on Spectroscopy

17. The Extension of Spectrophotometric Technique by Means of Special Type Cuvettes. DAVID L. DRABKIN, *School of Medicine, University of Pennsylvania.*

The applications of spectrophotometry have been considerably broadened by the design and use of two special cuvettes, a closed 3.4-cm depth microcuvette of capillary bore[1] and a cuvette of only 0.007-cm depth, provided with capillary entry and exit tubes.[2] (a) The 3.4-cm capillary cuvette has an internal diameter of 0.47 cm and chamber volume of 0.59 ml. The usual (Bausch and Lomb) 1-cm depth cuvette has an internal diameter of 1.7 cm and a capacity of 2.3 ml. The factors of depth and capacity operate together to increase sensitivity approximately 14-fold [3.4×(2.3/0.59)]. This permits direct spectrophotometry (hitherto not possible) to be applied to the determination of such substances as the biocatalyst cytochrome *c*, isolated from small amounts of normal or cancerous tissues[1,3] in which it may be present only in very low concentration. The adaptation of the capillary cuvette to the Bausch and Lomb polarization photometer, whose entrance pupils are considerably larger (0.8-cm diameter) than the bore of the cuvette, is accomplished by the use of a compensating iris diaphragm in front of one of the pupils. The diaphragm is a variable aperture, set to match extinction at the wave-length of minimal absorption for the solution measured. (b) The 0.007-cm cuvette has an optical chamber of 0.021-ml volume. The entry and exit capillaries connected with the chamber permit the cuvette to be filled with concentrated solutions, hemolyzed whole blood,[2,5] or non-hemolyzed whole blood,[4,5] without exposure to environmental gases. Solutions prepared from crystallized human hemoglobin, which is unusually soluble, have now been measured in this special cuvette at a concentration of 37.6 mM per liter (62.8 g per 100 ml), at *p*H 6.0. This is

two times the concentration of hemoglobin within the red blood cell, and by far the highest concentration recorded for transmission measurements upon the blood pigment.[3]

[1] O. Rosenthal and D. L. Drabkin, J. Biol. Chem. **149**, 437 (1943).
[2] D. L. Drabkin and J. H. Austin, J. Biol. Chem. **112**, 105 (1935–36).
[3] O. Rosenthal and D. L. Drabkin, J. Biol. Chem. **150**, 131 (1943); Am. J. Med. Sci. **204**, 914 (1942).
[4] D. L. Drabkin and R. B. Singer, J. Biol. Chem. **129**, 739 (1939).
[5] D. L. Drabkin and C. F. Schmidt, Am. J. Med. Sci. **208**, 133, 135 (1944).

18. Ultraviolet Absorption Methods of Analysis for Polyunsaturated Constituents of Fats, Oils, and Soaps. BROOKS A. BRICE AND MARGARET L. SWAIN, *Eastern Regional Research Laboratory,* Philadelphia, Pennsylvania.*

Ultraviolet spectrophotometric methods have recently been developed for determining polyunsaturated constituents in vegetable oils.[1–4] Difficulties were encountered in attempting to apply these methods to materials, such as animal fats and their soaps, that contain small proportions of polyunsaturated compounds. These difficulties have been largely overcome. A more accurate method for estimating conjugated constituents, involving corrections for extraneous background absorption, is presented. The spectrophotometric method of Mitchell, Kraybill, and Zscheile[1] for determining linoleic and linolenic acids, as extended by Beadle and Kraybill,[2] has been modified and improved for application to a wide variety of fats, oils, and preparations, including materials with small proportions of polyunsaturated compounds. Modifications include: the use of glycerol instead of ethylene glycol as an isomerization medium, which results in a more transparent and reproducible "blank"; elimination of filtering by use of absolute ethanol as a spectrophotometric solvent; correction of data for background and overlapping absorption; correction for conjugated constituents originally present; and use of simultaneous equations for calculation of results. Typical analyses are shown for linoleic, linolenic, and arachidonic acids and for conjugated diene, triene, and tetraene acids in tallows, tallow soaps, lard, vegetable oils, and purified fat acids. The accuracy of the method is discussed, and where possible, results are compared with those of chemical methods.

* One of the laboratories of the Bureau of Agricultural and Industrial Chemistry, Agricultural Research Administration, United States Department of Agriculture.
[1] Mitchell, Kraybill, and Zscheile, Ind. Eng. Chem. Anal. Ed. **15**, 1 (1943).
[2] Beadle and Kraybill, J. Am. Chem. Soc. **66**, 1232 (1944).
[3] Bradley and Richardson, Ind. Eng. Chem. **34**, 237 (1942).
[4] Brode, Patterson, Brown, and Frankel, Ind. Eng. Chem. Anal. Ed. **16**, 77 (1944).

19. Spectrochemical Analysis of Beryllium Ores and Commercial Products. LESTER W. STROCK, *Saratoga Springs Commission.*

Since presenting preliminary procedures for determining beryllium in beryllium minerals and chemical precipitates at the 1942 M.I.T. spectrographic conference, more accurate and generally applicable quantitative procedures have been devised for the rapid routine determination of beryllium in ores and commercial products. The methods have been extensively applied on several war production projects involving the analysis of several thousand samples. Details of the method will be presented, along with results of many experiments to test the effect of variation in composition and size of sample arced and nature of electrode material. Nickel was added as an internal intensity standard and contained in a magnetite-carbon powder buffer combined with each sample. The mixture so prepared was then used for preparing stepped spectra in the same manner previously described. The marked dependence of the intensity of Ni-3114 relative to Ni-3134 and Be-3130 on the electrode material is illustrated and its effect in the accuracy of the analysis will be emphasized. Quantitative procedures devised to determine impurities in beryllium oxide, beryllium copper alloy, and beryllium metal will also be outlined and illustrated by a few of the concentration calibration curves obtained on synthetic or analyzed standards. Finally, a new technique for determining major constituents of metallic alloy samples in a d.c. carbon arc will be illustrated by the determination of beryllium in beryllium copper alloy containing up to 5 percent Be. The same technique is applicable to other metals where small metallic samples may be analyzed. Use of this method enables laboratories equipped with d.c. arc equipment only, to analyze metals with the same, or better, accuracy and speed claimed for spark methods; and avoids the difficulties usually arising from arcing self-electrodes of metals. More complete data will be included in the printed, than in the presented, paper.

20. An Application of Spectrographic Methods to Chemical Concentrations of Trace Elements in Iron and Steel Analysis. R. A. WOLFE AND R. G. FOWLER, *University of Michigan.*

The work described was carried out for the Simonds Saw and Steel Company of Lockport, New York by the Department of Engineering Research at the University of Michigan. A combined spectrographic and chemical method for the analysis of trace elements in steels is discussed applicable to ranges in concentrations below the sensitivity of either the chemical or spectrograph methods. The investigations were undertaken because of the increasing recognition of the importance of minor constituents in the metallurgy of iron alloys. Details of the method are presented for the analysis of eighteen elements commonly found in iron alloys. The alloy samples were first treated by chemical extraction methods and the resulting concentrates analyzed spectrographically. Tables are given comparing chemical and spectrographic results at the higher concentrations where chemical analyses were attempted. The method described yields results of less accuracy than those obtained by the usual spectrographic methods but because of the low concentration involved is considered amply satisfactory for practical purposes.

21. Spectrographic Analysis of Wrought Copper Alloys. P. A. LEICHTLE AND A. F. WICHROWSKI, *Chase Brass and Copper Company, Waterbury, Connecticut.*

The spectrographic determination of trace elements[1] and minor constituents in a wide variety of wrought copper

alloys is discussed. Tables listing useful line pairs and approximate equal density values are given.

[1] P. A. Leichtle, J. Opt. Soc. Am. **34**, 454–463 (1944).

22. The Spectrochemical Analysis of Copper Base Alloys. EDWIN K. JAYCOX, *Bell Telephone Laboratories, Inc. Murray Hill, New Jersey.*

A method is described for the analysis of copper base alloys for aluminum, lead and tin (0.30 to 15.0 percent); iron, manganese, and nickel (0.30 to 5.0 percent); and zinc (0.30 to 40.0 percent). Samples are taken into solution with a mixture of nitric and hydrofluoric, or nitric and hydrochloric acids. The volume is adjusted so that the concentration of metals is 50.0 mg/ml. A portion of this solution is mixed with 19 parts of copper nitrate solution (copper = 50 mg/ml). Spectra are obtained of aliquots of each sample and of a series of standard solutions dried on porous graphite electrodes and excited in the d.c. arc. Determinations of the amounts of constituent elements present in the samples are made by measuring the ratio of intensities of a line of the constituent element to that of a copper control line, by the general internal standard procedure.

23. Ultraviolet Ionization of Interrupter Gaps in Spectrographic Spark Sources. S. F. SIMPSON, *Chevrolet Motor and Axle Division, General Motors Corporation.*

An ultraviolet lamp of high intensity is used to irradiate the synchronous interrupter in a conventional spectrographic spark source. A large reduction of the errors due to excitation was achieved on the commercial unit subjected to this test, but such improvement was negligible when tests were made on a spark unit of different design. The theory is presented that the improved uniformity of ionization is much less important to reproducibility of excitation than is the advancing of the oscillatory discharge in relation to the position of coincidence of the electrodes. This minimizes the irregularities caused by the periodicity or "hunting" of the driving motor.

BIBLIOGRAPHY

Vincent and Sawyer, J. Opt. Soc. Am. **32**, 686 (1942).
Allibone and Meek, J. Sci. Inst. **21** (Feb. 1944).
Kaiser and Wallraff, Ann. d. Physik **34**, 297 (1939).

24. A Non-Mechanical Interrupter for Controlled Spark Circuits. R. G. FOWLER AND R. A. WOLFE, *University of Michigan.*

A type of interrupted spark source involving no mechanical parts has been developed for use in quantitative spectrographic analysis. It possesses most of the desirable features of the mechanically interrupted spark sources, together with new characteristics of its own. It can be built with ease and economy. The essential part, an air jet and auxiliary spark gap, can be simply substituted for the mechanical interrupter on most any established spark circuit. Operation of the circuit has been accompanied by results of excellent precision. Commercial applications have proven satisfactory. The circuit should find application in a great many problems where spark circuits can be applied. Details of the circuit are given and results taken from actual operating conditions submitted.

Contributed Papers on General Optics

25. Preliminary Experiments in Phase Difference Microscopy. HELEN JUPNIK, *Spencer Lens Company, Buffalo, New York.*

Earlier investigators have found that it is possible, by suitably altering the phase of part of the light that has passed through the object, to improve the normal "positive" contrast and also to produce "negative" contrast in the image formed by a microscope of objects which differ from their surrounding medium mainly in refractive index. The effect produced on contrast in the image when both amplitude and phase changes are made in portions of the light passing through the objective has been studied by means of the following arrangement: Stops are placed in the focal plane of a centerable Abbe condenser such that the specimen is illuminated by light issuing from an annular opening. The stop system is itself centerable with respect to the condenser and is imaged in the rear focal plane of the objective. The phase and/or amplitude of the bundles of light corresponding to the bright and dark parts of the stop image is altered by placing in the focal plane of the objective a thin glass plate on which is evaporated suitable thicknesses of dielectric and/or metal both within and outside the region corresponding to the geometric image of the annular opening. A series of plates with different retardation and transmission factors was made for a 16-mm focus, N.A. 0.25 objective and a 4-mm focus, N.A. 0.66 objective, but in the case of the 1.8-mm focus, N.A. 1.25 oil immersion objective the material was evaporated on the most suitable lens surface. The 16-mm focus objective is provided with a rotary device which allows eight plates to be rotated successively into position. These coated plates and objectives have been used successfully with a large group of specimens having high transmission to produce increased positive or negative contrast. The series of plates point out that the introduction of absorption is important. Observations also indicate that the regions of discontinuity or of rapid variation of index of refraction in the object are the regions greatly enhanced in the image.

BIBLIOGRAPHY

A. Kohler and W. Loos, Naturwiss. **29**, 49 (1941); K. Michel, Naturwiss. **29**, 61 (1941).

26. Phase Difference Microscopy as a Problem in Diffraction. H. OSTERBERG, *Spencer Lens Company, Buffalo, New York.*

With the method of phase difference microscopy it is possible to enhance the contrast in the images of highly transparent objects. In order to produce favorable contrast

variations with an object of given size, light transmission, and optical path differences, it is desirable to predict the corresponding optimum distributions of light absorption and optical path differences in a plane suitably located at the microscope objective. A quite general theoretical attack upon the above and related problems will be outlined. In it, image formation is treated as a problem in diffraction based upon Huygen's principle. Slight contrast variations can be brought about in the images of self-luminous objects, but the more striking effects are possible with transparent objects in transmitted light. Theory and experiment show that the introduction of light absorbing means within the objective plays the major role in changing the state of contrast in the image. However, a series of interchangeable objectives or of plates in a given objective incorporating different distributions of both absorption and phase is required in order to produce the desired contrast variations with an arbitrary object specimen. Fortunately for the practical application of this method, a useful series for a given objective may consist of a small number, for example, seven, or fewer, properly selected distributions of absorption and phase. The method is applicable also to the modification of contrast in the images of objects of low light transmission. Theory and experiment associate greatest contrast differences in the case of highly transparent objects with those areas over which the optical path differences in the object vary rapidly. Large, uniform objects may exhibit marked contrast differences only in the vicinity of their contours. The effect of this method upon the resolution of a 16-mm microscope objective has been studied and found to be negligible for most of the distributions of absorption and phase which give excellent contrast differences with biological particles.

27. A Proposed Method of Specifying Appearance Defects of Optical Parts. J. H. McLEOD AND W. T. SHERWOOD, *Hawk-Eye Works, Eastman Kodak Company.*

A rather severe loss is encountered in the manufacture and assembly of optical glass parts because of accidental scratches or digs on the surfaces. It is important to be able to specify to what extent these defects are permitted in a given case. A system based on the comparison with standard samples of scratches and of digs has been devised. Blueprints carry two numbers that refer to the scratch and dig samples, respectively, and so specify the required quality line.

28. Coma-Free High Aperture Twin Ellipsoid Mirror. ROBERT A. WOODSON, *Eastman Kodak Company.*

An aplanatic optical system which forms a real image of a real object (not coinciding) at unit magnification for an arbitrarily large numerical aperture was desired in the design of an instrument for measuring reflectivity and transmissivity. These requirements, as well as freedom from chromatic aberration and ghost images, were satisfied by what is believed to be an original mirror system, the "twin ellipsoid" mirror. If it were not for the coma present in an ellipsoidal mirror, this well-known system would have sufficed, but—there was the coma![1] However, by combining two ellipsoids properly, coma can be eliminated. The twin ellipsoid consists of a pair of identical prolate ellipsoids of revolution placed with their major axes in line and with one focus in common. Only the portion of each ellipsoid outside the other is used. The object and image are at opposite foci. Absence of coma results from the over-all unit magnification for *all* rays starting at one extreme focus, reflecting once at each ellipsoid, and arriving at the other extreme focus. If m is the magnification for the first surface, then $1/m$ is the magnification for the second surface. The rays have the characteristic that the portion before the first reflection is parallel to the portion after the second reflection. This is approximately true also for objects having a finite size, so that the useful field is considerable even for very high numerical apertures. Thus a hollow cathode discharge tube can be imaged in a photoelectric tube with very high efficiency. Other applications for the Twin Ellipsoid mirror will help to evaluate it as a useful optical tool.

[1] There is a discussion of coma in Hardy and Perrin, *The Principles of Optics*, pages 91–96.

29. Rayleigh Interferometers for Gas Analysis. HENRY M. O'BRYAN, *Baird Associates.*

The Rayleigh interferometer for measuring refractive indices of gases has been modified to give a twofold increase in sensitivity without increasing the over-all length of the instrument. Instead of the usual two-compartment gas cell, a four-compartment cell is used. The upper left and lower right compartments are filled with the gas of unknown composition and the comparison gas is held in the upper right and lower left compartments. In this way the relative shift of the two fringe systems is twice that obtained with the conventional two-compartment gas cell. This enables one to measure smaller changes in refractive index or smaller variations in gas composition. Other features of this instrument are brilliant wide monochromatic fringes, a metal gas system, and convenient controls for the compensating plates and circulating system. A second portable instrument is especially designed for measuring the concentration of toxic gases in industrial atmospheres. The auto collimation principle with the four-compartment cell gives a 25-cm gas tube the same sensitivity as the conventional one-meter two-compartment gas cell. A small flashlight bulb and filter give bright fringes with sufficient contrast for accurate measurements.

30. Electro-Optical Effect in Colloids. B. W. SAKMANN, *Massachusetts Institute of Technology.*

The birefringence of colloidal solutions in alternating fields consists, in general, of a direct and a vibrating component both of which depend on field strength and frequency of the applied field, particle size, temperature, and concentration of the solution. The two components were separated in monodisperse bentonite sols by means of a photo-electric method.[1] The steady part of the birefringence

can be positive or negative and can reverse its sign with a change of field strength, frequency, particle size, temperature, and concentration. The measurements are interpreted by means of a relaxation theory which can explain the dependence of the vibrating component on the different parameters in a satisfactory way. Furthermore, the change and reversal of sign of the steady part of the double refraction with frequency and field strength can be explained by the assumption that the torque on the micelles is composed of a linear and a quadratic term. The torque which is directly proportional to the field orients the particles in such a way that the direction of greatest polarizability is perpendicular to the direction of the electric field, thus causing a negative birefringence. The torque which is proportional to the square of the field gives rise to a positive double refraction. The alterations and the reversal of sign of the birefringence with the temperature of the solutions and the diameter of the micelles can be accounted for by a change of the relaxation time of the oriented particles. The character of the dependence of the steady double refraction on concentration which has not hitherto been observed with other colloids was first discussed by H. Mueller.[2] The changes and reversal of sign of the double refraction with concentration are explained by the assumption that the micelles are surrounded by a compressed water hull. The pressure and the viscosity of the water hull and, therefore, the relaxation time of the micelles depend on particle size and concentration. This assumption is supported by measurements of Hauser and Le Beau[3] of the specific gravity of the suspended bentonite particles.

[1] H. Mueller and B. W. Sakmann, J. Opt. Soc. Am. **32**, 309 (1942).
[2] H. Mueller, Phys. Rev. **55**, 508, 792; **56**, 615 (1939).
[3] E. A. Hauser and D. S. Le Beau, J. Phys. Chem. **42**, 1031 (1933).

31. The Cos⁴ Law of Illumination. MAX REISS, *Eastman Kodak Company.*

In the design of photographic lenses, especially wide-angle lenses, it is of importance to know the theoretical light distribution over the image plane. Although the law governing this relationship is derivable from elementary principles (Lambert's law) there is a widespread discrepancy in the results given in various textbooks of optics. These discrepancies lead to paradoxical results which can be easily detected in simple cases like a lens with diaphragm in front or rear. Another example of special interest is the telecentric system used in all profile projection devices. In connection with a paper by Slussareff* the correct way of deriving the law of illumination is pointed out and the importance of the aberration of the pupils is discussed. Finally, the application of the law in practical cases is considered.

* G. Slussareft, J. Phys. U.S.S.R. **4**, No. 6, p. 537.

32. A Physical Photometer for Afocal Optical Systems. D. B. MCRAE, *Hawk-Eye Works, Eastman Kodak Company.*

A physical photometer is described which is particularly adapted to the measurement of the light transmission of any afocal optical systems. A collimated beam of "chopped" light provided by a projector, passes through the instrument whose transmission is to be measured,[1] and is picked up by a simple optical system which permits image brightness to be measured by a vacuum photo-tube and a.c. amplifier. The amplifier is designed to pass freely only frequencies of substantially over 200 cycles, so that the device is insensitive to daylight and 60-cycle artificial light. At the same time high sensitivity and good stability are obtained with regard to the light to be measured. Some methods of calibration are described. This photometer has proved useful in measuring transmission of large range finders as it can be taken to the range finder and measurements can be made in a well-lighted room.

[1] D. B. McRae, J. Opt. Soc. Am. **33**, 229 (1943).

33. Schlieren and Shadowgraph Equipment for Air Flow Analysis. NORMAN F. BARNES AND S. LAWRENCE BELLINGER, *General Engineering Laboratory, General Electric Company.*

The basic principle of schlieren equipment is described, showing how the optical system is set up to make temperature or pressure gradients visible in fluid media such as air. These variations can be either projected upon a screen for visual study, or upon a photographic plate. The merits of different types of light sources with their corresponding knife edges are given as well as orientations of the knife edge relative to the plane of major interest in the subject to be investigated. Circular and other types of light cutting edges can be used instead of the flat knife edge for application with various geometrical forms of the light source. From the standpoint of light output, simplicity of the optical system, and over-all sensitivity, the H-6, high pressure, 1000-watt mercury lamp with air cooling has been found to be the most satisfactory. This lamp has the additional advantage that besides its continuous operation for visual observation of schlieren phenomena, it can be combined with other devices, making an equipment called the electronic schlieren light source, and flashed for photographic recording of the schlieren pattern. Since the duration of a single flash is four microseconds or less, an effectively instantaneous picture can be made of air or particle velocities even in the supersonic field. The use of lenses *versus* concave mirrors as light collimators is shown. The effect of optical aberrations upon the system is discussed, with particular attention being given to spherical aberration and astigmatism. Shadowgraphic equipment is described for investigating phenomena accompanied by temperature or pressure gradients greater than that required for observation with schlieren apparatus. Though the H-6 lamp can again be used for continuous visual examination, the best results have been obtained using a high voltage spark discharge as a light source for photographic recording. This is accomplished with an electronic shadowgraph light source. Such a system requires only a photographic plate and the high intensity light source which subtends a very small solid angle when viewed from the plate.

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Minutes of the Twenty-Fifth Meeting of the Board of Directors of the Optical Society of America, Incorporated

HOTEL PENNSYLVANIA, NEW YORK, NEW YORK, OCTOBER 19, 1944

IN addition to the transaction of routine business, the following items of general interest to members of the Society were considered.

Announcement by Secretary

Result of letter ballot:

Approval of appointment of W. W. Graeper, Selig Hecht, David L. MacAdam, Daniel Smith, and John E. Tyler as members of the Program Committee for the 1944 annual meeting, under the chairmanship of A. Francis Turner.

Change in the Membership of the Board of Directors

The President announced the resignation of Wallace Brode as a member-at-large of the Board of Directors. The nomination of Norman Wright to fill the unexpired term of Wallace Brode was confirmed by a unanimous vote.

Report of the Committee on Honorary Memberships

The recommendation of the Committee on Honorary Memberships to confer honorary membership upon Professor R. W. Wood of the Johns Hopkins University was endorsed by a unanimous vote of the entire Board of Directors.

Election of Editor of Publications

George R. Harrison was elected to succeed himself as Editor of Publications for a term (four years) to expire on December 31, 1948.

Membership

The Secretary reported that between January 1 and October 7 there had been a net gain in membership of 89. On recommendation of the Committee on Transfer, 35 associate members were transferred to regular membership. The names of those transferred will appear in the Secretary's Annual Report for 1944.

Appointments

Nominating Committee for 1945 elections:

A. G. Worthing, Chairman
W. E. Forsythe
L. A. Jones
G. S. Monk
W. B. Rayton.

Ives Medal Committee:

W. F. Little, Chairman
D. L. MacAdam
G. W. Moffitt
E. D. Tillyer
George Wald.

Program Committee for Spring Meeting, 1945:

Norman Wright, Chairman.

O.S.A. Representatives on the Inter-Society Color Council, for a term to expire in October, 1946:

R. C. Gibbs,* Chairman
I. H. Godlove*
D. B. Judd*
E. Q. Adams
Blanche Bellamy
W. C. Granville
A. C. Hardy
L. A. Jones
Parry Moon
Dorothy Nickerson.

* Voting delegates.

O.S.A. Representative on the Council of the American Association for the Advancement of Science, for a term to expire in February, 1946:

G. H. Dieke.

Nomination

O.S.A. Representative on the Governing Board of the American Institute of Physics, for a term to expire in February, 1948:

K. S. Gibson.

ARTHUR C. HARDY, *Secretary*

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